Online Appendices

A Theoretical Appendix

A.1 Assumptions

Assumption 1. *Let $\mathcal{I}_t$ denote the information set at time t and $\mathbb{Q}_\tau(y_{t+1}|\mathcal{I}_t)$ denote the time-t conditional $\tau-$quantile of y_{t+1}. Let f_t be 1×1 and $\boldsymbol{g}_t$ be $K_g \times 1$ with $K = 1+K_g$, $\boldsymbol{F}_t \equiv (f_t, \boldsymbol{g}_t')'$, and $\boldsymbol{x}_t$ be $N\times 1$, for $t = 1,\dots,T$. Then*

1. $\mathbb{Q}_\tau(y_{t+1}|\mathcal{I}_t) = \mathbb{Q}_\tau(y_{t+1}|\boldsymbol{f}_t) = \alpha_0 + \boldsymbol{\alpha}(\tau)'\boldsymbol{F}_t = \alpha_0 + \alpha(\tau) f_t$
2. $y_{t+1} = \alpha_0 + \alpha(\tau) f_t + \eta_{t+1}(\tau)$
3. $\boldsymbol{x}_t = \boldsymbol{\lambda}_0 + \boldsymbol{\phi} f_t + \boldsymbol{\Psi}\boldsymbol{g}_t + \boldsymbol{\varepsilon}_t = \boldsymbol{\lambda}_0 + \boldsymbol{\Lambda}\boldsymbol{F}_t + \boldsymbol{\varepsilon}_t$

where $\boldsymbol{\Lambda} \equiv (\boldsymbol{\lambda}_1,\dots,\boldsymbol{\lambda}_N)'$.

Assumption 2. *Let $||\boldsymbol{A}|| = (tr(\boldsymbol{A}'\boldsymbol{A}))^{1/2}$ denote the norm of matrix $\boldsymbol{A}$, and M be some positive finite scalar.*

1. *The variables $\{\Lambda_i\}$, $\{\boldsymbol{F}_t\}$, $\{\varepsilon_{it}\}$ and $\{\eta_{it}\}$ are independent groups.*
2. $\mathbb{E}||\boldsymbol{F}_t||^4 \le M < \infty$ *and* $\frac{1}{T}\sum_{t=1}^T \boldsymbol{F}_t\boldsymbol{F}_t' \to \boldsymbol{\Sigma}_F$ *or some $K\times K$ positive definite matrix* $\boldsymbol{\Sigma}_F \equiv \begin{bmatrix} \Sigma_f & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\Sigma}_g \end{bmatrix}$.
3. $||\lambda_i|| \le \bar{\lambda} < \infty$ *and* $||\boldsymbol{\Lambda}'\boldsymbol{\Lambda}/N - \boldsymbol{\Sigma}_\Lambda|| \to 0$ *for some $K\times K$ positive definite matrix* $\boldsymbol{\Sigma}_\Lambda \equiv \begin{bmatrix} \Sigma_\phi & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\Sigma}_\psi \end{bmatrix}$.
4. *For all (i,t),* $\mathbb{E}(\varepsilon_{it}) = 0, \mathbb{E}|\varepsilon_{it}|^8 \le M$
5. *There exist* $\mathbb{E}(\varepsilon_{it}\varepsilon_{js}) = \sigma_{ij,ts}$ *and* $|\sigma_{ij,ts}| < \bar{\sigma}_{ij}$ *for all (t,s), and* $|\sigma_{ij,ts}| \le \tau_{ts}$ *for all (i,j) such that* $\frac{1}{N}\sum_{i,j=1}^N \bar{\sigma}_{ij} \le M$, $\frac{1}{T}\sum_{t,s=1}^T \tau_{ts} \le M$, *and* $\frac{1}{NT}\sum_{i,j,s,t=1} |\sigma_{ij,ts}| \le M$
6. *For every (t,s),* $\mathbb{E}|\frac{1}{\sqrt{N}}\sum_{i=1}^N [\varepsilon_{is}\varepsilon_{it} - \mathbb{E}(\varepsilon_{is}\varepsilon_{it})]|^4 \le M$

Assumption 3. *Let m, M be positive finite scalars. For each $\tau \in (0,1)$ the shock $\eta_{t+1}(\tau)$ has conditional density $\pi_\tau(\cdot|\mathcal{I}_t) \equiv \pi_{\tau t}$ and is such that*

1. *$\pi_{\tau t}$ is everywhere continuous*
2. *$m \le \pi_{\tau t} \le M$ for all t*
3. *$\pi_{\tau t}$ satisfies the Lipschitz condition $|\pi_{\tau t}(\kappa_1) - \pi_{\tau t}(\kappa_2)| \le M|\kappa_1 - \kappa_2|$ for all t*

Assumption 4. *Let M be a positive finite scalar.*

1. *In addition to Assumption 2.1, $\{f_t\}$ is independent of $\{\boldsymbol{g}_t\}$ and $\{\phi_i\}$ is independent of $\{\boldsymbol{\psi}_i\}$*

2. $\{\varepsilon_{it}\}$ *are i.i.d.*

3. $\lim_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N}\frac{1}{\phi_i} < M$.

4. $\mathbb{E}(f_t^n)$, $\mathbb{E}(\boldsymbol{g}_t^n)$ *and* $\mathbb{E}(\boldsymbol{\varepsilon}_t^n)$ *exist and are finite for all* n.

5. $\{\boldsymbol{g}_t\}$ *and* $\{\boldsymbol{\psi}_i\}$ *have symmetric distributions.*

Proof Outline Assumptions 1 and 2 are the same as those in Bai and Ng's (2006) work on principal components factor estimates in OLS regressions. Adding assumption 3 is sufficient to show that quantile regression is consistent in a time series setting, because the assumptions imply Engle and Manganelli's (2004) assumptions C0-C7 and AN1-AN4, which they show satisfy Corollary 5.12 of White (1994). Assumption 4 strengthens some moment and independence conditions of Assumption 2 and additionally imposes conditions on the distributions of ϕ_i, $\boldsymbol{\psi}_i$ and $\boldsymbol{g}_u$.

Our approach views the latent factor structure among systemic risk measures as an errors-in-variables quantile regression problem. To address this, we rely heavily on mis-specified quantile regression results from Angrist, Chernozhukov and Fernandez-Val (2006, ACF hereafter) to express biases that arise in population for various stages of the PCQR and PQR procedures.[30]

For PCQR, Bai (2003) tells us that the principal component factor estimates converge to a rotation of the true factor space at rate $\min(\sqrt{N}, T)$ under Assumptions 1 and 2. We write an infeasible second stage quantile regression of y_{t+1} on the factor estimate and its deviation from the true factor. The probability limit of this infeasible quantile regression follows by Assumption 3 and allows for an ACF bias representation of the feasible quantile regression of y_{t+1} on the factor estimate alone. This allows us to show that the fitted conditional quantile from the second stage quantile regression is consistent for the true conditional quantile for N, T large.

The proof for PQR looks similar. The main difference is PQR's latent factor estimator, which is not based on PCA. PQR's first stage quantile regressions of y_{t+1} on x_{it} involves an errors-in-variables bias that remains in the large N and T limit. We write an infeasible first stage quantile regression of y_{t+1} on x_{it} and the two components of its measurement error $(\boldsymbol{g}_t, \varepsilon_{it})$. For each i, the probability limit of this infeasible quantile regression follows by Assumptions 1-3 and allows for an ACF bias representation of the feasible quantile regression regression of y_{t+1} on x_{it} alone. For each t, the factor estimate comes from cross-sectional covariance of x_{it} with the mis-measured first-stage coefficients. This converges to a scalar times the true factor at rate $\min(\sqrt{N}, \sqrt{T})$ under Assumption 4. This results makes use of a fact about the covariance of a symmetrically-distributed random variable with a rational function of its square, which is proved in Lemma 1. The third stage quantile regression using this factor is consistent for the true conditional quantile in the joint N, T limit, by following the argument in the proof for PCQR.

[30] The results of Bai (2003) and Bai and Ng (2008a) can be used to establish the consistency of the PCQR. Alternatively, one could deduce the consistency from Ando and Tsay's (2011) consistency proof for an information criteria using a PCQR model. We provide an alternative derivation in order to closely connect the proofs of both PCQR and PQR.

A.2 Proof of Theorem 1

Proof. Let $\hat{\boldsymbol{F}}_t$ be given by the first K principal components of $\boldsymbol{x}_t$. Bai (2003) Theorem 1 implies that for each t, $\hat{\boldsymbol{F}}_t - \boldsymbol{H}\boldsymbol{F}_t$ is at least $O_p(\delta_{NT}^{-1})$, where $\delta_{NT} \equiv \min(\sqrt{N}, \sqrt{T})$, $\boldsymbol{H} = \tilde{\boldsymbol{V}}^{-1}(\tilde{\boldsymbol{F}}'\boldsymbol{F}/T)(\boldsymbol{\Lambda}'\boldsymbol{\Lambda}/N)$, $\tilde{\boldsymbol{F}} \equiv (\tilde{\boldsymbol{F}}_1, \ldots, \tilde{\boldsymbol{F}}_T)$ is the matrix of K eigenvectors (multiplied by $\sqrt{T}$) associated with the K largest eigenvalues of $\boldsymbol{X}\boldsymbol{X}'/(TN)$ in decreasing order, and $\tilde{\boldsymbol{V}}$ is the $K \times K$ diagonal matrix of the K largest eigenvalues.[31]

The second stage quantile regression coefficient is given by

$$(\hat{\alpha}_0, \hat{\boldsymbol{\alpha}}) = arg \min_{\alpha_0, \boldsymbol{\alpha}} \frac{1}{T} \sum_{t=1}^{T} \rho_\tau(y_{t+1} - \alpha_0 - \boldsymbol{\alpha}'\hat{\boldsymbol{F}}_t).$$

Consider an infeasible regression of y_{t+1} on the PCA factor estimate $\hat{\boldsymbol{F}}_t$ as well as the factor estimation error $\hat{\boldsymbol{F}}_t - \boldsymbol{H}\boldsymbol{F}_t$ (for given N and T). Because $\boldsymbol{F}_t$ linearly depends on $(\hat{\boldsymbol{F}}_t, \hat{\boldsymbol{F}}_t - \boldsymbol{H}\boldsymbol{F}_t)$, this regression nests the correctly specified quantile forecast regression. By White (1994) Corollary 5.12 and the equivariance properties of quantile regression we have that the infeasible regression coefficients

$$(\dot{\alpha}_0, \dot{\boldsymbol{\alpha}}, \dot{\boldsymbol{\alpha}}_1) = arg \min_{\alpha_0, \boldsymbol{\alpha}_1, \boldsymbol{\alpha}} \frac{1}{T} \sum_{t=1}^{T} \rho_\tau(y_{t+1} - \alpha_0 - \boldsymbol{\alpha}'\hat{\boldsymbol{F}}_t - \boldsymbol{\alpha}_1'(\hat{\boldsymbol{F}}_t - \boldsymbol{H}\boldsymbol{F}_t)),$$

are such that $\dot{\boldsymbol{\alpha}}$ satisfies

$$\sqrt{T}(\dot{\boldsymbol{\alpha}} - \boldsymbol{\alpha}'\boldsymbol{H}^{-1}) \xrightarrow[T \to \infty]{d} N(\boldsymbol{0}, \boldsymbol{\Sigma}_{\dot{\boldsymbol{\alpha}}}).$$

Next, ACF (2006) Theorem 1 implies that

$$\hat{\boldsymbol{\alpha}} = \dot{\boldsymbol{\alpha}} + \left(\sum_{u=1}^{T} w_u \hat{\boldsymbol{F}}_u \hat{\boldsymbol{F}}_u' \right)^{-1} \left(\sum_{u=1}^{T} w_u \hat{\boldsymbol{F}}_u \dot{\boldsymbol{\alpha}}_1' (\hat{\boldsymbol{F}}_u - \boldsymbol{H}\boldsymbol{F}_u) \right) \tag{A1}$$

where they derive the weight function $w_t = \frac{1}{2} \int_0^1 \pi_\tau \left(v \left[\hat{\boldsymbol{\alpha}}' \hat{\boldsymbol{F}}_t - \alpha f_t \right] \right) dv$.

Next, we rewrite the forecast error as

$$\hat{\boldsymbol{\alpha}}'\hat{\boldsymbol{F}}_t - \boldsymbol{\alpha}'\boldsymbol{F}_t = \hat{\boldsymbol{\alpha}}'(\hat{\boldsymbol{F}}_t - \boldsymbol{H}\boldsymbol{F}_t) + (\hat{\boldsymbol{\alpha}}' - \boldsymbol{\alpha}'\boldsymbol{H}^{-1})\boldsymbol{H}\boldsymbol{F}_t. \tag{A2}$$

As stated above, the first term of (A2) is no bigger than $O_p(\delta_{NT}^{-1})$. To evaluate the second term, use (A1) to obtain

$$(\hat{\boldsymbol{\alpha}}' - \boldsymbol{\alpha}'\boldsymbol{H}^{-1}) = (\dot{\boldsymbol{\alpha}}' - \boldsymbol{\alpha}'\boldsymbol{H}^{-1}) + \left(\frac{1}{T} \sum_{u=1}^{T} w_u \hat{\boldsymbol{F}}_u \hat{\boldsymbol{F}}_u' \right)^{-1} \left(\frac{1}{T} \sum_{u=1}^{T} w_u \hat{\boldsymbol{F}}_u \dot{\boldsymbol{\alpha}}_1' (\hat{\boldsymbol{F}}_u - \boldsymbol{H}\boldsymbol{F}_u) \right). \tag{A3}$$

[31] Bai (2003) shows that $\hat{\boldsymbol{F}}_t - \boldsymbol{H}\boldsymbol{F}_t$ is $O_p(\min(\sqrt{N}, T)^{-1})$, which is at least as fast a rate of convergence as $O_p(\min(\sqrt{N}, \sqrt{T})^{-1})$.

The first term on the right-hand side is $O_p(T^{-1/2})$, as stated above. Use $\hat{\boldsymbol{F}}_u \equiv \hat{\boldsymbol{F}}_u - \boldsymbol{H}\boldsymbol{F}_u + \boldsymbol{H}\boldsymbol{F}_u$ to rewrite the numerator of the second term on the right-hand side

$$\frac{1}{T}\sum_{u=1}^{T} w_u \hat{\boldsymbol{F}}_u \dot{\boldsymbol{\alpha}}_1'(\hat{\boldsymbol{F}}_u - \boldsymbol{H}\boldsymbol{F}_u)$$
$$= \delta_{NT}^{-2}\frac{1}{T}\sum_{u=1}^{T} w_t \delta_{NT}(\hat{\boldsymbol{F}}_u - \boldsymbol{H}\boldsymbol{F}_u)\dot{\boldsymbol{\alpha}}_1'\delta_{NT}(\hat{\boldsymbol{F}}_u - \boldsymbol{H}\boldsymbol{F}_u) + \delta_{NT}^{-1}\frac{1}{T}\sum_{u=1}^{T} w_t \boldsymbol{H}\boldsymbol{F}_u \dot{\boldsymbol{\alpha}}_1'\delta_{NT}(\hat{\boldsymbol{F}}_u - \boldsymbol{H}\boldsymbol{F}_u)$$
$$= \delta_{NT}^{-2}O_p(1) + \delta_{NT}^{-1}O_p(1).$$

Therefore the right-hand side of (A3) is $O_p(T^{-1/2}) + O_p(1)O_p(\delta_{NT}^{-1}) = O_p(\delta_{NT}^{-1})$. This implies that $\hat{\boldsymbol{\alpha}}' - \boldsymbol{\alpha}'\boldsymbol{H}^{-1}$ is $O_p(\delta_{NT}^{-1})$. Putting this back into (A2), we see therefore that $\hat{\boldsymbol{\alpha}}'\hat{\boldsymbol{F}}_t - \boldsymbol{\alpha}'\boldsymbol{F}_t$ is $O_p(1)O_p(\delta_{NT}^{-1}) + O_p(\delta_{NT}^{-1})O_p(1) = O_p(\delta_{NT}^{-1})$ which completes the result. □

A.3 Proof of Theorem 2

Proof. For each i, the first stage quantile regression coefficient is given by

$$(\hat{\gamma}_{0i}, \hat{\gamma}_i) = arg \min_{\gamma_0,\gamma} \frac{1}{T}\sum \rho_\tau(y_{t+1} - \gamma_0 - \gamma x_{it}). \tag{A4}$$

Consider the infeasible quantile regression of y_{t+1} on $(x_{it}, \boldsymbol{g}_t', \varepsilon_{it})'$, yielding coefficient estimates

$$(\dot{\gamma}_{0i}, \dot{\gamma}_i, \dot{\boldsymbol{\gamma}}_{ig}', \dot{\gamma}_{i\varepsilon})' = arg \min_{\gamma_0,\gamma,\boldsymbol{\gamma}_g,\gamma_\varepsilon} \frac{1}{T}\sum_{t=1}^{T} \rho_\tau(y_{t+1} - \gamma_0 - \gamma x_{it} - \boldsymbol{\gamma}_g'\boldsymbol{g}_t - \gamma_\varepsilon \varepsilon_{it}).$$

Note that f_t linearly depends on the vector $(x_{it}, \boldsymbol{g}_t', \varepsilon_{it})'$. By White (1994) Corollary 5.12 and the equivariance properties of quantile regression, these coefficients satisfy

$$\sqrt{T}(\dot{\gamma}_i, \dot{\boldsymbol{\gamma}}_{ig}', \dot{\gamma}_{i\varepsilon})' \xrightarrow[T\to\infty]{d} N\left(\left(\frac{\alpha}{\phi_i}, -\frac{\alpha}{\phi_i}\boldsymbol{\psi}_i', -\frac{\alpha}{\phi_i}\right)', \boldsymbol{\Sigma}_\gamma\right)$$

ACF (2006) Theorem 1 implies that

$$\hat{\gamma}_i = \dot{\gamma}_i + \left(\sum_{u=1}^{T} w_{iu} x_{iu}^2\right)^{-1}\left(\sum_{u=1}^{T} w_{iu} x_{iu}\left(\dot{\boldsymbol{\gamma}}_{ig}'\boldsymbol{g}_u + \dot{\gamma}_{i\varepsilon}\varepsilon_{iu}\right)\right). \tag{A5}$$

for the weight $w_{it} = \frac{1}{2}\int_0^1 (1-u)\pi_\tau\left(u\left[x_{it}\hat{\gamma}_i - \mathbb{Q}\left(y_{t+1}|f_t\right)\right]|f_t\right)du$.[32] Expanding the weight

[32]This weight comes from the fact that in our factor model the true conditional quantile $\mathbb{Q}(y_{t+1}|\mathcal{I}_t)$ is identical to the quantile conditioned only on f_t. In addition, the conditioning of π_τ on f_t is a choice of representation and consistent with our assumption that no other time t information influences the distribution of η_{t+1}. ACF provide a detailed derivation of this weight as a function of the quantile forecast error density, which they denote as f rather than π.

around $x_{it} = 0$, we have

$$w_{it} = \sum_{n=1}^{\infty} \kappa_n(f_t) x_{it}^n \quad , \quad \kappa_n(f_t) \equiv \frac{1}{n!} \left. \frac{\partial^n w_{it}}{(\partial x_{it})^n} \right|_{x_{it}=0} \tag{A6}$$

and can use this to rewrite (A5). Note that $\kappa_n(f_t)$ is a function *only* of f_t and is therefore independent of $\boldsymbol{g}_t, \varepsilon_{it}$. Also note that $x_{it}^n = \sum_{j=0}^{n} (\phi_i f_t)^{n-j} (\boldsymbol{\psi}' \boldsymbol{g}_t + \varepsilon_{it})^j a_{n,j}$, where the $a_{n,j}$'s are polynomial expansion coefficients. Using the following notation

$$\Gamma_1 = \left(\sum_{u=1}^{T} w_{iu} x_{iu}^2 \right)^{-1} \left(\sum_{u=1}^{T} w_{iu} x_{iu} \left[(\dot{\boldsymbol{\gamma}}_{ig} + \frac{\alpha}{\phi_i} \boldsymbol{\psi}_i)' \boldsymbol{g}_u + (\dot{\gamma}_{i\varepsilon} + \frac{\alpha}{\phi_i}) \varepsilon_{iu} \right] \right),$$

$$\Gamma_2 = -\frac{\alpha}{\phi_i} \left(\frac{1}{T} \sum_{u=1}^{T} w_{iu} x_{iu}^2 \right)^{-1} \left(\sum_{n=0}^{\infty} \sum_{j=0}^{n+1} a_{n+1,j} \left[\frac{1}{T} \sum_{u=1}^{T} \kappa_n(f_u) (\phi_i f_u)^{n+1-j} (\boldsymbol{\psi}_i' \boldsymbol{g}_u + \varepsilon_{iu})^{j+1} \right. \right.$$
$$\left. \left. - \mathbb{E} \left(\kappa_n(f_u) (\phi_i f_u)^{n+1-j} (\boldsymbol{\psi}_i' \boldsymbol{g}_u + \varepsilon_{iu})^{j+1} \right) \right] \right),$$

$$\Gamma_3 = -\frac{\alpha}{\phi_i} \times \left(\sum_{n=0}^{\infty} \sum_{j=0}^{n+1} a_{n+1,j} \mathbb{E} \left(\kappa_n(f_u) (\phi_i f_u)^{n+1-j} (\boldsymbol{\psi}_i' \boldsymbol{g}_u + \varepsilon_{iu})^{j+1} \right) \right)$$
$$\left(\sum_{n=0}^{\infty} \sum_{j=0}^{n+2} a_{n+2,j} \left[\mathbb{E} \left(\kappa_n(f_u) (\phi_i f_u)^{n+2-j} (\boldsymbol{\psi}_i' \boldsymbol{g}_u + \varepsilon_{iu})^j \right) - \frac{1}{T} \sum_{u=1}^{T} \kappa_n(f_u) (\phi_i f_u)^{n+2-j} (\boldsymbol{\psi}_i' \boldsymbol{g}_u + \varepsilon_{iu})^j \right] \right)^{-1},$$

$$\Gamma_4 = \dot{\gamma}_i - \frac{\alpha}{\phi_i},$$

we can rewrite (A5) as

$$\hat{\gamma}_i = \frac{\alpha}{\phi_i} - \frac{\alpha}{\phi_i} \frac{\sum_{n=0}^{\infty} \sum_{j=0}^{n+1} a_{n+1,j} \mathbb{E} \left[\kappa_n(f_t) (\phi_i f_t)^{n+1-j} \right] \sum_{k=0}^{j+1} a_{j+1,k} \mathbb{E} \left[(\boldsymbol{\psi}_i' \boldsymbol{g}_t)^{j+1-k} \right] \mathbb{E} \left[\varepsilon_{it}^k \right]}{\sum_{n=0}^{\infty} \sum_{j=0}^{n+2} a_{n+2,j} \mathbb{E} \left[\kappa_n(f_t) (\phi_i f_t)^{n+2-j} \right] \sum_{k=0}^{j} a_{j,k} \mathbb{E} \left[(\boldsymbol{\psi}_i' \boldsymbol{g}_t)^{j-k} \right] \mathbb{E} \left[\varepsilon_{it}^k \right]}$$
$$+ \Gamma_1 + \Gamma_2 + \Gamma_3 + \Gamma_4. \tag{A7}$$

Because of the probability limit noted above for $(\dot{\gamma}_i, \dot{\boldsymbol{\gamma}}_{ig}', \dot{\gamma}_{i\varepsilon})'$, we know that Γ_1 and Γ_4 are $O_p(T^{-1/2})$. Γ_2 and Γ_3 are also $O_p(T^{-1/2})$ by Assumption 4, the continuous mapping theorem, and the law of large numbers. By Assumption 4, for any i and for n odd we have $\mathbb{E}\left[(\boldsymbol{\psi}_i' \boldsymbol{g}_t)^n\right] = 0$. Therefore we can rewrite the above expression for $\hat{\gamma}_i$ as

$$\hat{\gamma}_i = \frac{\alpha}{\phi_i} - \frac{\alpha}{\phi_i} \Upsilon(\boldsymbol{\psi}_i^2, \phi_i) + O_p(T^{-1/2})$$

where Υ is the rational function given by the second term in A7. We write Υ as a function of $\boldsymbol{\psi}_i^2$ and ϕ_i because we have integrated out the dependence on $f, \boldsymbol{g}, \varepsilon_i$ using the expectation operator.

The second stage factor estimate is[33]

$$\hat{f}_t = \frac{1}{N}\sum_{i=1}^{N}\left(\hat{\gamma}_i - \bar{\hat{\gamma}}\right)\left(x_{it} - \bar{x}_t\right)$$
$$= \frac{1}{N}\sum_{i=1}^{N}\left(\frac{\alpha}{\phi_i} - \frac{\alpha}{\phi_i}\Upsilon(\boldsymbol{\psi}_i^2,\phi_i) + O_p(T^{-1/2}) - \bar{\hat{\gamma}}\right)\left((\phi_i - \bar{\phi})f_t + (\boldsymbol{\psi}_i - \bar{\boldsymbol{\psi}})'\boldsymbol{g}_t + (\varepsilon_{it} - \bar{\varepsilon}_t)\right).$$

What we need is that the parts of $\hat{f}_t$ involving $\boldsymbol{g}_t$ and ε_{it} vanish as $N, T \to \infty$. Sums involving ε_{it} vanish as N becomes large by the independence of ε_{it} and $(\phi_i, \boldsymbol{\psi}_i')'$. Now consider all the terms involving $\boldsymbol{g}_t$. The term involving $\boldsymbol{g}_t$ multiplied by $N^{-1}\sum_i(\frac{\alpha}{\phi_i} - \frac{\bar{\alpha}}{\phi_i})(\boldsymbol{\psi}_i - \bar{\boldsymbol{\psi}})'$ vanishes from $\hat{f}_t$ for N large due to the independence of $\phi_i, \boldsymbol{\psi}_i$. Then the term involving cross products of $\Upsilon(\boldsymbol{\psi}_i^2, \phi_i)$ and $(\boldsymbol{\psi}_i - \bar{\boldsymbol{\psi}})'$ vanish in probability as N becomes large by the symmetry of $\boldsymbol{\psi}_i$ (Assumption 4) and Lemma 1. The remaining terms are of smaller stochastic order.

Then straightforward algebra shows that $\hat{f}_t - hf_t$ is at least $O_p(\delta_{NT}^{-1})$, where is h a finite nonzero constant.[34] From here, Theorem 1's argument applies, starting from the paragraph involving (A2). □

Lemma 1. *For any symmetrically-distributed random variable x, random vector $\boldsymbol{y} = (y_1, ..., y_{d-1})$ such that $x \perp \boldsymbol{y}$, and rational function $f : \mathcal{R}^d \to \mathcal{R}^1$ that is infinitely differentiable at some number $\boldsymbol{a} \in \mathcal{R}^d$, it is the case that $Cov(f(x^2, \boldsymbol{y}), x) = 0$.*

Proof. Define the vector $\boldsymbol{x} = (x^2, \boldsymbol{y}')'$, so that $x_1 = x^2$ and $x_j = y_{j-1}$. The Taylor series for $f(\boldsymbol{x})$ at $\boldsymbol{a}$ is

$$f(a_1,\ldots,a_d) + \sum_{j=1}^{d}\frac{\partial f(a_1,\ldots,a_d)}{\partial x_j}(x_j - a_j) + \frac{1}{2!}\sum_{j=1}^{d}\sum_{k=1}^{d}\frac{\partial^2 f(a_1,\ldots,a_d)}{\partial x_j \partial x_k}(x_j - a_j)(x_k - a_k) +$$
$$+\frac{1}{3!}\sum_{j=1}^{d}\sum_{k=1}^{d}\sum_{l=1}^{d}\frac{\partial^3 f(a_1,\ldots,a_d)}{\partial x_j \partial x_k \partial x_l}(x_j - a_j)(x_k - a_k)(x_l - a_l) + \ldots$$

Any cross products involving x_j for $j > 1$ have zero covariance with x by independence. By the symmetry of x, $Cov(x_1^i, x) = 0$ for any $i = 0, 1, ...$, which proves the result. □

A.4 Simulation Evidence

Table A7 compares PCQR and PQR estimates with the true 0.1 conditional quantile. We report the time series correlation between the true conditional quantile and

[33] Overbar denotes a sample mean over i.

[34] It can be shown that

$$N^{-1}\sum_{i=1}^{N}\left(\Upsilon(\boldsymbol{\psi}_i^2,\phi_i) - \overline{\Upsilon(\boldsymbol{\psi}^2,\phi)}\right)(\phi_i - \bar{\phi})$$

converges to a finite constant that is different from one, which implies that h is nonzero.

the fitted series as well as the time series mean absolute error (MAE) averaged over simulations. The simulated model is

$$\begin{aligned} y_{t+1} &= -f_t \mathbb{1}_L + (\sigma_\eta + f_t \mathbb{1}_S)\, \eta_{t+1} \\ \boldsymbol{x}_t &= \boldsymbol{\phi} f_t + \boldsymbol{\psi} g_t + \boldsymbol{e}_t \end{aligned}$$

We draw $f \sim U(0,1)$, $g \sim N(0, 0.5^2)$, $e_{it} \sim N(0, 0.5^2)$, $\eta \sim N(0, 0.5^2)$, $\phi_i \sim N(0, 0.5^2)$, and $\psi_i \sim N(0, 0.5^2)$, all independent. We pick $\mathbb{1}_L = 1$ for a location model and $\mathbb{1}_L = 0$ otherwise, $\mathbb{1}_S = 1$ for a scale model and $\mathbb{1}_S = 0$ otherwise, and $\mathbb{1}_L = \mathbb{1}_S = 1$ for a location and scale model. We vary T, set $N = T$, and run 1,000 simulations of each specification. The table reports performance of quantile forecasts from PCQR using two principal component indexes and from PQR using a single index. It shows that conditional quantile forecasts are increasingly accurate in the size of the predictor panel. As N and T grow, the time series correlation between fits and the true conditional quantile approaches one and the forecast error shrinks toward zero.

B Empirical Appendix

B.1 Systemic Risk Measures

CoVaR and ΔCoVaR (Adrian and Brunnermeier (2011)) CoVaR is defined as the value-at-risk (VaR) of the financial system as a whole conditional on an institution being in distress. The distress of the institution, in turn, is captured by the institution being at its own individual VaR (computed at quantile q):

$$Pr(X^i < \text{VaR}^i) = q$$

CoVaR for institution i is then defined as:

$$Pr(X^{syst} < \text{CoVaR}^i | X^i = \text{VaR}^i) = q$$

which we estimate using conditional linear quantile regression after estimating VaR^i. ΔCoVaR^i is defined as the VaR of the financial system when institution i is at quantile q (in distress) relative to the VaR when institution i is at the median of its distribution:

$$\Delta\text{CoVaR}^i = \text{CoVaR}^i(q) - \text{CoVaR}^i(0.5).$$

In estimating CoVaR, we set q to the 5th percentile. Note that Adrian and Brunnermeier (2011) propose the use of a conditional version of CoVaR as well, called *forward* CoVaR, in which the relation between the value-at-risk of the system and an individual institution is allowed to depend on an additional set of covariates. Here we use the alternative approach of rolling window CoVaR estimates with an estimation window of 252 days. We construct individual CoVaR for each firm separately and calculate the aggregate measure as an equal-weighted average among the largest 20 financial firms.

MES (Acharya, Pedersen, Philippon and Richardson (2010)) These measures capture the exposure of each individual firm to shocks to the aggregate system. MES captures the expected return of a firm conditional on the system being in its lower tail:

$$\text{MES}^i = E[R^i | R^m < q]$$

where q is a low quantile of the distribution of R_m (we employ the 5th percentile). We construct individual MES for each firm separately using a rolling window of 252 days and calculate the aggregate measure as an equal-weighted average among the largest 20 financial firms.

MES-BE (Brownlees and Engle (2011)) This version of MES employs dynamic volatility models (GARCH/DCC for $\sigma_{\cdot,t}, \rho_t$) to estimate the components of MES:

$$\text{MES-BE}_{i,t-1} = \sigma_{i,t}\rho_t E\left[\epsilon_{m,t} | \epsilon_{m,t} < \frac{k}{\sigma_{m,t}}\right] + \sigma_{i,t}\sqrt{1-\rho_t^2} E\left[\epsilon_{i,t} | \epsilon_{m,t} < \frac{k}{\sigma_{m,t}}\right].$$

where $\epsilon_{m,t}$ are market return shocks, $\epsilon_{i,t}$ is the individual firm return and k is set to 2 following Brownlees and Engle (2011). We construct the measure individually for each firm and calculate the aggregate measure as an equal-weighted average among the largest 20 financial firms.

CatFin (Allen, Bali and Tang (2012)) This measure computes the time-varying value at risk (VaR) of financial institutions at the 99% confidence level, using the cross-sectional distribution of returns on the equity of financial firms in each period. In particular, the methodology first fits (parametrically or nonparametrically) a distribution for the lower tail (bottom 10%) of the cross-sectional distribution of returns of financial institutions, separately in each month. CatFin is then obtained as the 1st percentile of returns under the fitted distribution, computed separately in each month.

Allen et al. (2012) propose computing the VaR by fitting two types of parametric distributions, the Generalized Pareto Distribution (GPD) and the Skewed Generalized Error Distribution, as well as nonparametrically using the empirical cross-sectional distribution of returns (simply computing in each month the 1st percentile of the returns realized across firms in that month), and then averaging the three measures to construct CatFin.

In our implementation of CatFin (which differs slightly from the specification in Allen et al. (2012) for consistency with the other measures we build), we construct the measure at the monthly frequency by pooling together all daily returns of the top 20 financial firms in each month, and using them to estimate the tail distribution and compute the 1st percentile of returns. Given the extremely high correlation (above 99%) among the three ways of computing the VaR (already noted by Allen et al. (2012)), we use the nonparametric version of CatFin obtained using the empirical distribution of returns.

Absorption Ratio (Kritzman et al. (2010)) This measure computes the fraction of return variance of a set of N financial institutions explained by the first $K < N$ principal components:

$$\text{Absorption}(K) = \frac{\sum_{i=1}^{K} Var(PC_i)}{\sum_{i=1}^{N} Var(PC_i)}.$$

A leading distress indicator is then constructed as the difference between absorption ratios calculated for long and short estimation windows

$$\Delta\text{Absorption}(K) = \text{Absorption}(K)_{\text{short}} - \text{Absorption}(K)_{\text{long}}.$$

In our empirical analysis we construct the Absorption(3) measure using returns for the largest 20 financial institutions at each point in time. We construct ΔAbsorption(3) using 252 and 22 days for the long and short windows, respectively.

Dynamic Causality Index or DCI (Billio et al. 2012) The index aims to capture how interconnected a set of financial institutions is by computing the fraction of significant Granger-causality relationships among their returns:

$$\text{DCI}_t = \frac{\#\, significant\, GC\, relations}{\#\, relations}$$

A Granger-causality relation is defined as significant if its p-value falls below 0.05. We construct the measure using daily returns of the largest 20 financial institutions, with a rolling window of 36 months.

International Spillover (Diebold and Yilmaz 2009) The index, downloaded from http://economicresearchforum.org/en/bcspill, aggregates the contribution of each variable to the forecast error variance of other variables across multiple return series. It captures the total extent of spillover across the series considered (a measure of interdependence).

Volatility We construct individual volatility series of financial institutions by computing the within-month standard deviation of daily returns. We construct the aggregated series of volatility by averaging the individual volatility across the 20 largest institutions.

Book and Market Leverage We construct a measure of aggregate book leverage (debt/assets) and aggregate market leverage (debt/market equity) among the largest 20 financial institutions to capture the potential for instability and shock propagation that occurs when large intermediaries are highly levered.

Size Concentration We construct the Herfindal index of the size distribution among financial firms:

$$\text{Herfindahl}_t = N\frac{\Sigma_{i=1}^{N} ME_i^2}{(\Sigma_{i=1}^{N} ME_i)^2}$$

The concentration index captures potential instability due to the threat of default of the largest firms. The index corrects for the changing number of firms in the sample by multiplying the measure of dispersion by the number of firms, N. When constructing this measure we use the market equity of the largest 100 firms.

Turbulence (Kritzman and Li (2010)) Turbulence is a measure of excess volatility that compares the realized squared returns of financial institutions with their historical volatility:

$$\text{Turbulence}_t = (r_t - \mu)'\Sigma^{-1}(r_t - \mu)$$

where r_t is the vector of returns of financial institutions, and μ and Σ are the historical mean and variance-covariance matrix. We compute the moments using data for the largest 20 financial institutions and a rolling window of 60 months.

AIM (Amihud 2002) AIM captures a weighted average of stock-level illiquidity AIM_t^i, defined as:

$$\text{AIM}_t^i = \frac{1}{K}\sum_{\tau=t-K}^{t}\frac{|r_{i,\tau}|}{turnover_{i,\tau}}$$

We construct an aggregated measure by averaging the measure across the top 20 financial institutions.[35]

TED Spread The difference between three-month LIBOR and three-month T-bill interest rates.

Default Yield Spread The difference between yields on BAA and AAA corporate bonds. The series is computed by Moody's and is available from the Federal Reserve Bank of St. Louis.

Gilchrist-Zakrajsek Spread Gilchrist and Zakrajsek (2012) propose an alternative measure of credit spread constructed from individual unsecured corporate bonds, where the yield of each bond is compared to the yield of a synthetic treasury bond with the same cash flows to obtain a precise measure of its credit spread. The individual credit spreads are then averaged across all maturities and all firms to obtain an index, GZ. We obtained the series from Simon Gilchrist's website.

Term Spread The difference between yields on the ten year and the three month US Treasury bond. The series is obtained from Global Financial Data.

[35] Our definition of AIM differs from that of Amihud (2002). We replace dollar volume with share turnover to avoid complications due to non-stationarity.

B.2 Macroeconomic Shocks

Let the monthly macroeconomic series (CFNAI or IP growth) be denoted Y_t. We construct shocks to these series as residuals in an autoregression of the form

$$Y_t = c + \sum_{l=1}^{p} a_l Y_{t-l} = c_p + a_p(L) Y_t$$

for a range of autoregressive orders, p, and select the p that minimizes the Akaike Information Criterion. This approach purges each macroeconomic variable of predictable variation based on its own lags, and is a convention in the macro forecasting literature (e.g. Bai and Ng (2008b) and Stock and Watson (2012)).

Shocks are estimated in a recursive out-of-sample scheme to avoid look-ahead bias in our out-of-sample quantile forecasting tests. For each month τ, we estimate the AR and AIC on data only known through τ, and construct the forecast residual at time $\tau + 1$ based on these estimates. Finally, we construct quarterly shocks as a moving three-month sum of the monthly residuals.

B.3 In-Sample Statistics

The in-sample R^2 lies between zero and one. In sample, we report the statistical significance of the predictive coefficients as found by Wald tests (or t-statistics for univariate regressions) using standard errors from the residual block bootstrapped with block lengths of six months and 1,000 replications.

Table A3 Panel A reports the quantile R^2 from in-sample 20^{th} percentile forecasts of IP growth shocks in the US, UK and EU using the collection of systemic risk measures. Our main analysis uses data from 1946-2011 for the US, 1978-2011 for the UK, and 1994-2011 for the EU. In sample, a wide variety of systemic risk measures demonstrate large predictive power for the conditional quantiles for IP growth shocks in various countries. This picture changes when we look out-of-sample.

Panel B of Table A3 shows that joint use of many systemic risk measures produces a high in-sample R^2 when predicting the 20^{th} percentile of future IP growth shocks in the US, UK and EU. The table shows that Multiple QR (that simultaneously includes all the systemic risk variables) works best by this metric. But Table 2 Panel B illustrates the expected results of in-sample overfit: Multiple QR's out-of-sample accuracy is extremely poor.

B.4 Quantile Granger Causality Tests

An alternative to the pre-whitening procedure described in Appendix B.2 is to control for the history each dependent variable within the quantile regression specification, as in an in-sample Granger causality test. This alternative procedure yields qualitatively similar results to those reported in the main text.

To conduct a Granger causality test in our framework, consider the quantile regression

$$\mathbb{Q}_\tau(Y_t|\mathcal{I}_t) = \beta_0 + \sum_{l=1}^{p} \beta_p Y_{t-p} + \sum_{k=1}^{q} \gamma_k x_{t-k}$$

where Y is monthly IP growth and x is a systemic risk measure. We investigate whether x Granger causes the quantiles of Y by testing the hypothesis: $\gamma_1 = \cdots = \gamma_q = 0$. We estimate the standard error matrix of $(\boldsymbol{\beta}', \boldsymbol{\gamma}')'$ using Politis and Romano's (1994) stationary block-bootstrap with 1,000 bootstrap replications and choose $q = 1$. Table A8 reports the resulting Wald statistics for the 20^{th} percentile and median, each of which is asymptotically distributed as a $\chi^2(1)$.

B.5 Interval Coverage Tests

An alternative method of evaluating the quantile forecasts follows Christoffersen (1998). We take the quantile forecast $\hat{q}$ to define the interval $(-\infty, \hat{q})$ and evaluate this interval's coverage. Christoffersen (1998) provides likelihood ratio tests for the intervals' correct conditional coverage. Table A9 reports the resulting likelihood ratio tests using the 20^{th} percentile.

Table A1: Correlations Among Systemic Risk Measures

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
Panel A: US																				
Absorption	(1)	1.00																		
AIM	(2)	-0.01	1.00																	
CoVaR	(3)	0.60	0.20	1.00																
ΔCoVaR	(4)	0.68	0.06	0.95	1.00															
MES	(5)	0.63	0.14	0.92	0.93	1.00														
MES-BE	(6)	0.35	-0.09	0.38	0.41	0.47	1.00													
Book Lvg.	(7)	0.24	-0.06	0.13	0.10	0.10	-0.06	1.00												
CatFin	(8)	0.36	0.33	0.59	0.48	0.53	0.33	0.11	1.00											
DCI	(9)	0.13	-0.07	0.34	0.36	0.39	0.28	0.08	0.23	1.00										
Def. Spr.	(10)	0.25	0.33	0.67	0.53	0.55	0.34	-0.25	0.56	0.24	1.00									
ΔAbsorption	(11)	-0.51	-0.02	-0.25	-0.28	-0.31	-0.15	-0.05	0.13	-0.02	-0.06	1.00								
Intl. Spillover	(12)	0.42	-0.13	0.40	0.45	0.45	0.25	0.12	0.19	0.17	0.34	-0.15	1.00							
GZ	(13)	0.73	-0.12	0.75	0.71	0.71	0.36	0.33	0.62	0.26	0.37	-0.23	0.31	1.00						
Size Conc.	(14)	0.04	0.28	0.34	0.18	0.26	0.00	0.40	0.29	0.14	0.36	-0.04	-0.07	0.45	1.00					
Mkt Lvg.	(15)	-0.14	0.11	0.22	0.19	0.17	-0.09	0.30	0.24	0.51	0.45	0.13	0.29	0.15	0.00	1.00				
Real Vol.	(16)	0.38	0.24	0.71	0.59	0.64	0.44	0.13	0.88	0.28	0.61	0.07	0.19	0.69	0.29	0.19	1.00			
TED Spr.	(17)	0.10	0.05	0.19	0.20	0.20	0.34	-0.34	0.48	0.12	0.38	0.02	-0.16	0.24	-0.2	0.09	0.49	1.00		
Term Spr.	(18)	0.29	0.01	0.35	0.37	0.33	0.34	-0.22	0.12	0.20	0.40	-0.12	0.31	0.16	0.09	-0.08	0.14	-0.07	1.00	
Turbulence	(19)	0.13	-0.05	0.20	0.18	0.18	0.22	0.1	0.42	0.12	0.16	0.02	0.06	0.41	0.02	0.17	0.48	0.54	-0.06	1.00
Panel B: UK																				
Absorption	(1)	1.00																		
CoVaR	(2)	0.56	1.00																	
ΔCoVaR	(3)	0.68	0.97	1.00																
MES	(4)	0.60	0.92	0.93	1.00															
MES-BE	(5)	0.43	0.48	0.53	0.65	1.00														
CatFin	(6)	0.30	0.64	0.61	0.62	0.61	1.00													
DCI	(7)	0.40	0.33	0.37	0.45	0.39	0.19	1.00												
ΔAbsorption	(8)	-0.48	-0.30	-0.36	-0.33	-0.12	0.15	-0.22	1.00											
Size Conc.	(9)	0.01	0.25	0.23	0.41	0.51	0.32	0.27	0.01	1.00										
Real Vol.	(10)	0.35	0.69	0.66	0.67	0.68	0.94	0.21	0.13	0.35	1.00									
Turbulence	(11)	0.14	0.41	0.38	0.39	0.48	0.66	0.04	0.04	0.15	0.70	1.00								
Panel C: EU																				
Absorption	(1)	1.00																		
CoVaR	(2)	0.65	1.00																	
ΔCoVaR	(3)	0.75	0.95	1.00																
MES	(4)	0.76	0.94	0.96	1.00															
MES-BE	(5)	0.49	0.46	0.61	0.59	1.00														
CatFin	(6)	0.20	0.35	0.26	0.30	0.09	1.00													
DCI	(7)	0.44	0.55	0.58	0.59	0.42	0.19	1.00												
ΔAbsorption	(8)	-0.51	-0.32	-0.37	-0.40	-0.24	0.30	-0.21	1.00											
Size Conc.	(9)	-0.01	0.21	0.19	0.10	0.00	-0.17	0.20	-0.10	1.00										
Real Vol.	(10)	0.31	0.56	0.50	0.50	0.31	0.84	0.34	0.20	-0.04	1.00									
Turbulence	(11)	0.03	0.13	0.11	0.10	0.16	0.30	0.13	0.08	-0.07	0.43	1.00								

Notes: Correlation is calculated using the longest available coinciding sample for each pair.

Table A2: Pairwise Granger Causality Tests

	US		UK		EU	
	Causes	Caused by	Causes	Caused by	Causes	Caused by
Absorption	7	4	2	1	1	5
AIM	1	3	-	-	-	-
CoVaR	9	5	6	4	4	4
ΔCoVaR	7	7	4	5	3	4
MES	6	10	5	7	3	6
MES-BE	3	11	5	9	1	6
Book Lvg.	0	0	-	-	-	-
CatFin	10	10	6	6	3	4
DCI	1	7	0	8	3	0
Def. Spr.	9	4	-	-	-	-
ΔAbsorption	4	0	5	0	4	0
Intl. Spillover	0	8	-	-	-	-
GZ	8	1	-	-	-	-
Size Conc.	1	0	1	0	0	0
Mkt Lvg.	2	0	-	-	-	-
Real Vol.	10	6	7	3	7	5
TED Spr.	5	1	-	-	-	-
Term Spr.	1	10	-	-	-	-
Turbulence	7	4	7	5	6	1

Notes: For each pair of variables, we conduct two-way Granger causality tests. The table reports the number of other variables that each measure significantly Granger causes (left column) or is caused by (right column) at the 2.5% one-sided significance level (tests are for positive causation only). Tests are based on the longest available coinciding sample for each pair.

Table A3: In-Sample 20^{th} Percentile IP Shock Forecasts

	US	UK	EU
	Panel A: Individual Systemic Risk Measures		
Absorption	0.10	2.20**	8.38***
AIM	3.75***	0.56	0.67
CoVaR	3.07***	4.95***	7.20***
ΔCoVaR	1.27***	4.45***	7.96***
MES	1.53***	3.28***	6.86***
MES-BE	0.14	2.32**	6.11***
Book Lvg.	1.06	0.27	0.32
CatFin	5.65***	4.87***	11.47***
DCI	0.14*	0.44	7.09***
Def. Spr.	2.11***	9.95***	15.04***
ΔAbsorption	0.18**	0.11	0.42
Intl. Spillover	0.55**	1.58***	2.36*
GZ	8.05***	5.06***	19.44***
Size Conc.	0.04	0.77**	3.02**
Mkt. Lvg.	10.42***	0.76**	12.21***
Volatility	3.81***	8.00***	12.65***
TED Spr.	7.73***	6.61***	8.30***
Term Spr.	1.65**	0.07	3.08***
Turbulence	3.85***	2.43***	5.46***
	Panel B: Systemic Risk Indexes		
Multiple QR	32.69	22.89	41.40
Mean	0.20	1.99***	9.03***
PCQR1	13.24***	11.30***	16.28***
PCQR2	17.91***	12.50***	18.24***
PQR	18.44***	10.93***	11.55***

Notes: The table reports in-sample quantile forecast R^2 (in percentage) relative to the historical quantile model. Statistical significance at the 10%, 5% and 1% levels are denoted by *, ** and ***, respectively; we do not test the Multiple QR model. Sample is 1946-2011 for US data, 1978-2011 for UK data, and 1994-2011 for EU data. Rows "Absorption" through "Turbulence" use each systemic risk measure in a univariate quantile forecast regression for the IP growth shock of the region in each column. "Multiple QR" uses all systemic risk measures jointly in a multiple quantile regression. Rows "Mean" through "PQR" use dimension reduction techniques on all the systemic risk measures. Mean is a simple average, PCQR1 and PCQR2 use one and two principal components, respectively, in the PCQR forecasting procedure, while PQR uses a single factor.

Table A4: 10^{th} Percentile IP Shock Forecasts

Out-of-sample start:	1950	1976	1990
	Panel A: Individual Systemic Risk Measures		
Absorption	−2.98	−9.93	−9.26
AIM	6.41***	3.12	6.02*
CoVaR	−0.62	−0.07	−1.02
ΔCoVaR	−1.34	−1.48	−1.40
MES	−2.14	−0.51	0.62
MES-BE	−2.56	−4.82	−17.19
Book Lvg.	–	7.22***	3.31***
CatFin	5.48***	12.63***	15.40***
DCI	0.56	2.65	4.31*
Def. Spr.	0.67	3.58***	6.96***
ΔAbsorption	−1.91	−0.27	−0.36
Intl. Spillover	–	6.51**	8.22***
GZ	–	6.92**	16.46***
Size Conc.	−2.19	−7.56	−3.24
Mkt. Lvg.	–	18.68***	18.94***
Volatility	2.99*	5.28*	4.94
TED Spr.	–	–	11.45**
Term Spr.	1.13	4.53**	−1.91
Turbulence	2.32	8.01**	12.74**
		Panesk Indexes	
Multiple QR	−114.56	−63.29	−6.86
Mean	−5.12	−11.00	−22.95
PCQR1	−1.10	2.71	−1.57
PCQR2	0.51	10.07**	9.90*
PQR	5.07*	16.54***	15.48***

Notes: The table reports out-of-sample quantile forecast R^2 (in percentage) relative to the historical quantile model. Statistical significance at the 10%, 5% and 1% levels are denoted by *, ** and ***, respectively; we do not test the Multiple QR model. Sample is . In-sample statistics are in column one. The out-of-sample start is noted for columns two through four. Rows "Absorption" through "Turbulence" use each systemic risk measure in a univariate quantile forecast regression for US IP growth rate shocks. "Multiple QR" uses all systemic risk measures jointly in a multiple quantile regression. Rows "PCQR1" through "PQR" use dimension reduction techniques on all the systemic risk measures. Mean is a simple average, PCQR1 and PCQR2 use one and two principal components, respectively, in the PCQR forecasting procedure, while PQR uses a single factor. "–" indicates insufficient data for estimation in a given sample.

Table A5: 10^{th} Percentile CFNAI Shock Forecasts

	Total	PH	PI	SOI	EUH
	Panel A: Individual Systemic Risk Measures				
Absorption	−7.19	−5.13	−6.84	−8.79	−5.21
AIM	−7.75	−4.35	−8.19	−5.56	−4.43
CoVaR	−6.12	−1.13	−4.87	−2.32	−0.64
ΔCoVaR	−6.53	−1.16	−7.53	−5.10	−4.17
MES	−8.13	−2.46	−10.35	−6.55	−4.97
MES-BE	−5.30	−3.13	−3.64	−4.12	−4.52
Book Lvg.	−3.52	−3.08	−1.71	−0.76	1.30
CatFin	5.72	1.26	5.72	5.22	9.52*
DCI	−2.69	−1.63	−0.68	−2.67	−1.57
Def. Spr.	−1.04	−3.82	−1.48	−0.78	−0.05
ΔAbsorption	0.21	−5.05	−0.85	1.87	0.74
Intl. Spillover	−6.30	−3.85	−3.44	−3.23	−2.07
GZ	−12.13	−6.08	−11.19	−11.99	−8.17
Size Conc.	−4.04	−2.97	−2.21	−6.14	−0.64
Mkt. Lvg.	8.74**	4.95**	2.68	4.39	4.13
Volatility	−2.06	−3.51	−1.68	1.30	2.08
TED Spr.	6.29*	7.67**	5.66	11.47**	−1.32
Term Spr.	1.40	−2.58	0.42	0.97	1.02
Turbulence	13.41***	5.08*	14.65**	11.64***	9.63**
	Panel B: Systemic Risk Indexes				
Multiple QR	−104.76	−109.39	−98.97	−74.59	−86.96
Mean	0.80	1.16	−0.09	2.21	−7.10
PCQR1	−9.73	−2.38	−9.70	−5.75	−3.16
PCQR2	−4.09	−2.93	−2.41	−1.54	1.01
PQR	7.29*	1.79	8.21*	7.33*	8.15*

Notes: The table reports out-of-sample quantile forecast R^2 (in percentage) relative to the historical quantile model. Statistical significance at the 10%, 5% and 1% levels are denoted by *, ** and ***, respectively; we do not test the Multiple QR model. Sample is 1967-2011. Out-of-sample period starts in 1976, except for Ted Spread which begins later. Rows "Absorption" through "Turbulence" use each systemic risk measure in a univariate quantile forecast regression for the CFNAI index or sub-index in each column. "Multiple QR" uses all systemic risk measures jointly in a multiple quantile regression. Rows "PCQR1" through "PQR" use dimension reduction techniques on all the systemic risk measures. Mean is a simple average, PCQR1 and PCQR2 use one and two principal components, respectively, in the PCQR forecasting procedure, while PQR uses a single factor.

Table A6: 80^{th} Percentile IP Shock Forecasts

	IP	CFNAI
Panel A: Individual Systemic Risk Measures		
Absorption	1.14	0.01
AIM	3.57^{***}	-3.97
CoVaR	0.76	-2.52
ΔCoVaR	0.40	-2.97
MES	-1.29	-3.12
MES-BE	0.33	-0.53
Book Lvg.	–	-1.32
CatFin	0.19	-2.78
DCI	-3.96	-0.80
Def. Spr.	-4.48	-3.38
ΔAbsorption	-0.92	-1.85
Intl. Spillover	–	-0.75
GZ	–	-2.34
Size Conc.	-3.02	-0.59
Mkt. Lvg.	–	0.23
Volatility	0.34	-2.69
TED Spr.	16.22^{***}	7.41^{***}
Term Spr.	-4.14	-4.28
Turbulence	0.31	-0.28
Panel B: Systemic Risk Indexes		
Multiple QR	-48.21	-68.59
Mean	4.69^{**}	0.44
PCQR1	-3.47	-2.39
PCQR2	-6.00	-8.74
PQR	-5.73	0.24

Notes: The table reports out-of-sample quantile forecast R^2 (in percentage) relative to the historical quantile model. Statistical significance at the 10%, 5% and 1% levels are denoted by *, ** and ***, respectively; we do not test the Multiple QR model. Sample is 1946-2011 for IP and 1967-2011 for CFNAI. Out-of-sample period starts in 1976, except for Ted Spread which begins later. Rows "Absorption" through "Turbulence" use each systemic risk measure in a univariate quantile forecast regression for IP growth shocks. "Multiple QR" uses all systemic risk measures jointly in a multiple quantile regression. Rows "PCQR1" through "PQR" use dimension reduction techniques on all the systemic risk measures. Mean is a simple average, PCQR1 and PCQR2 use one and two principal components, respectively, in the PCQR forecasting procedure, while PQR uses a single factor.

Table A7: Simulation Evidence

	Location		Scale		Loc. and Scale	
T, N	Corr.	MAE	Corr.	MAE	Corr.	MAE
			Panel A: PCQR			
$T, N = 50$	0.87	0.61	0.76	2.77	0.89	0.50
$T, N = 100$	0.94	0.33	0.85	6.39	0.95	0.28
$T, N = 500$	0.99	0.12	0.98	0.16	0.99	0.11
$T, N = 1,000$	0.99	0.08	0.99	0.11	1.00	0.07
			Panel B: PQR			
$T, N = 50$	0.74	0.80	0.56	3.07	0.72	0.90
$T, N = 100$	0.84	0.51	0.70	1.06	0.84	0.54
$T, N = 500$	0.96	0.22	0.91	0.33	0.96	0.21
$T, N = 1,000$	0.98	0.15	0.95	0.22	0.98	0.15

Notes: Simulation evidence using the model described in the text. We consider dimensions for T, N between 50 and 1,000. We report time series correlation and mean absolute pricing error between the true and estimated 0.1 conditional quantiles. Panel A reports results for PCQR using two principal component indexes, and Panel B reports results for PQR using a single index. The simulated model is described in Appendix A.

Table A8: In-Sample Granger Causality Tests

	20^{th}	Median
Absorption	0.04	2.92*
AIM	74.54***	0.00
CoVaR	11.16***	12.10***
ΔCoVaR	8.33***	5.30**
MES	6.04**	4.46**
MES-BE	0.03	0.00
Book Lvg.	0.24	0.00
CatFin	40.74***	10.86***
DCI	0.14	0.13
Def. Spr.	8.77***	20.51***
ΔAbsorption	0.03	0.08
Intl. Spillover	0.21	1.77
GZ	6.24**	20.07***
Size Conc.	0.01	0.00
Mkt. Lvg.	10.37***	7.19***
Volatility	12.48***	23.19***
TED Spr.	4.65**	0.04
Term Spr.	1.02	1.11
Turbulence	12.64***	6.64***

Notes: The table reports Wald statistics of the test that the systemic risk measure (by row) does not Granger cause (in the quantile sense) IP growth in the regression at a particular quantile (by column). Statistical significance at the 10%, 5% and 1% levels are denoted by *, ** and ***, respectively; we do not test the Multiple QR model. Sample period is 1946-2011. Rows "Absorption" through "Turbulence" use each systemic risk measure (by row) singly in a quantile regression.

Table A9: Conditional Coverage Tests of the Intervals defined by the 20^{th} Percentile, for IP Shocks

	1950	1970	1990
Absorption	0	0	0
AIM	0	0	0
CoVaR	0	0	$***$
ΔCoVaR	0	0	$***$
MES	0	$***$	$***$
MES-BE	0	0	0
Book Lvg.	0	0	0
CatFin	0	0	0
DCI	0	0	0
Def. Spr.	0	0	0
ΔAbsorption	0	0	0
Intl. Spillover	0	0	0
GZ	0	0	$***$
Size Conc.	0	0	0
Mkt. Lvg.	0	0	0
Volatility	0	$**$	$***$
TED Spr.	$-$	$-$	0
Term Spr.	0	0	0
Turbulence	0	0	0
Multiple QR	0	0	0
MEAN	0	0	0
PCQR1	0	0	$***$
PCQR2	0	0	$***$
PQR	0	0	0

Notes: The table reports likelihood ratio test significant of the null hypothesis that the estimated quantile $\hat{q}$ defines an interval $(-\infty, \hat{q})$ that has correct conditional coverage, following Christoffersen (1998). Statistical significance at the 10%, 5% and 1% levels are denoted by *, ** and ***, respectively; acceptance of the null hypothesis is denoted by "0".