A Review of Long-Run Discounting: Evidence From Housing Markets

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Abstract

We review the work of Giglio, Maggiori and Stroebel (2015, 2016) (GMS15,16) and Giglio, Maggiori, Stroebel and Weber (2015) (GMSW15) on long run discounting and bubbles. They explore how households trade off immediate costs and uncertain future benefits that occur in the very long run, 100 or more years away. They exploit a unique feature of housing markets in the U.K. and Singapore, where residential property ownership takes the form of either leaseholds or freeholds. Leaseholds are temporary, pre-paid, and tradable ownership contracts with maturities between 99 and 999 years, while freeholds are perpetual ownership contracts. The price difference between leaseholds and freeholds reflects the value of a claim to the freehold after the leasehold expires and is informative of long-run discount rates and the possible presence of bubbles. They estimate the price discounts for varying leasehold maturities relative to freeholds and extremely long-run leaseholds, using hedonic regressions with data on the universe of housing transactions in each country. GMS15 find that households discount very long-run housing cash flows at low rates, assigning high present value to cash flows hundreds of years in the future. GMS16 develop a test for rational bubbles and rule out their presence in the U.K. and Singapore housing markets for the period 1995-2013. GMSW15 estimate the entire term structure of housing discount rates and combine it with estimates of housing riskiness and structural models to infer how households trade off risk and return at extremely long horizons; they bring this unique setup to bear on the debate regarding the appropriate discount rate for climate change abatement investments.

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1 Introduction

In this article we review and complement the work on very long-run discounting by Giglio, Maggiori and Stroebel (2015, 2016) and Giglio, Maggiori, Stroebel and Weber (2015). In what follows we refer to these papers in brief as GMS15, GMS16, and GMSW15. This body of work has shown how a seemingly quirky feature of housing markets in the U.K. and Singapore, the presence of both leasehold and freehold ownership contracts for residential real estate, can be exploited to learn about how households discount potentially uncertain payoffs that occur hundreds of years into the future and to test for the presence of rational bubbles. Combining these observations with structural models offers guidance on how households trade off risk and return over long horizons.

Long-run discount rates play a central role in economics and public policy. For example, much of the debate around the optimal response to climate change centers on the trade-off between the immediate costs and the very long-term benefits of policies that aim to reduce global warming (Nordhaus, 2007; Weitzman, 2007; Gollier, 2006; Barro, 2013). Similar cost-benefit analyses are necessary in many governmental decisions, because of their intergenerational nature.

Unfortunately, direct empirical evidence on how households discount payments over very long horizons has been lacking, because of the scarcity of finite, long-maturity assets necessary to estimate households' valuation of very long-run claims. For regulatory action with "intergenerational benefits or costs," the U.S. Office of Management and Budget therefore recommends a wide range of discount rates (1% - 7%), lamenting that while "private markets provide a reliable reference for determining how society values time within a generation, for extremely long time periods no comparable private rates exist."

GMS15 provide direct estimates of households' discount rates for payments very far into the future (100 years and beyond). They exploit a unique feature of residential housing markets in the U.K. and Singapore, where property ownership takes the form of either long-term leaseholds or freeholds. Leaseholds are temporary, pre-paid and tradable ownership contracts with maturities ranging from 99 to 999 years, while freeholds are perpetual ownership contracts. The price difference between leaseholds and freeholds for otherwise identical properties captures, in the absence of rational bubbles, the present value of perpetual rental income starting at leasehold expiry, and is thus informative about households' discount rates over that horizon. GMS15 estimate the discount rates for these housing cash flows to be approximately 2.6% per year for horizons of 100 or more years.

GMS16 focus on extremely-long leaseholds (maturity greater than 700 years) and freeholds and show that their price difference is informative of the possible presence of rational bubbles. Bubbles of this type may arise on infinite maturity assets since the asset can be traded infinitely many times, at potentially higher and higher prices. The theory of these bubbles was established in seminal contributions by Samuelson (1958); Diamond (1965); Tirole (1982, 1985). Subsequently, rational bubbles have become the workhorse model of bubbles in macroeconomics (e.g., Caballero and Krishnamurthy, 2006; Arce and López-Salido, 2011; Martin, 2012; Martin and Ventura, 2014; Farhi and Tirole, 2012; Doblas-Madrid, 2012; Giglio and Severo, 2012; Gali, 2014; Galí and Gambetti, 2014; Caballero and Farhi, 2014). In these models, each trader purchases the asset purely due to the expectation of being able to resell it in the future at a sufficiently high price, even if the price today is above the fundamental value. Such bubbles cannot arise on finite maturity assets since the terminal period breaks the infinite loop of price-increase expectations. At maturity, the price has to collapse to its fundamental value, and backward induction then implies that the price has to always equal the fundamental value in all previous periods. GMS16 test for the presence of bubbles by comparing the prices of extremely-long (close to 1000 years of maturity) leaseholds and freeholds. These two contracts have the same fundamental present value, since they differ only by cash flows occurring more than 700 years into the future, but are differentially affected by the presence of rational bubbles since these bubbles would only increase the value of the freehold. They find that extremely-long leaseholds and freeholds are priced identically, thus ruling out the presence of rational bubbles in the U.K. and Singapore housing markets for the period 1995-2013.

Finally, in ongoing work, GMSW15 provide new evidence on the entire term structure of housing discount rates and the riskiness of housing cash flows, and show how these data can be combined with structural models to inform important economic decisions, such as climate change policy, that rely crucially on long run discount rates. GMS15 and GMS16 focused only on long-run discount rates (100 year of maturity and above) and did not study the shape of the entire term structure, nor decomposed discount rates into their risk free and risk premium components. Studying the entire term structure of discount rates, and understanding the maturity dependence of risk and return, are important steps in extracting information from one particular asset (housing) to understand more generally long-run discounting for other investments (for example, investments to mitigate climate change). GMSW15 find the term structure of discount rates for housing to be downward sloping with long run cash flows being discounted at substantially lower rates than short run cashflows. This result on long run term structure is complementary to recent empirical work documenting downward sloping term structures of discount rates in equity markets up to 10 year maturity (van Binsbergen et al., 2012, 2013) and in other markets like the market for variance risk (Dew-Becker et al., 2016; van Binsbergen and Koijen, 2016).

Section 2 provides an elementary review of discounting, Sections 3 and 4 review the institutional features of leaseholds and freeholds and discuss how to use their prices to learn about long-run discounting, Section 5 develops a convenient reduced-form declining discount function, Section 6 reviews the results in GMS15, Section 7 reviews the results in GMS16, and Section 8 reviews the results in GMSW15.

2 Discounting: The Role of Risk and Horizon

We start by reviewing the main concepts relating to long-run discounting, and the role of maturity and risk premia. The review in this section follows closely GMSW15; we refer the reader to that paper for additional details.

The analysis concerns the discounting of a stream of stochastic cash flows arising at different times in the future, D_{t+k}, $k = 1, 2..., T$, where T is the final maturity of the cash flows. Under no arbitrage, the time-t price of such stream of cash flows, P_t, is equal to the sum of expected dividends adjusted for risk and time through an appropriate Stochastic Discount Factor (SDF) ξ:

$$P_t = \sum_{k=1}^{T} E_t[\xi_{t,t+k} D_{t+k}].$$

A cash flow that occurs at maturity $t+k$ is discounted using the maturity-specific SDF $\xi_{t,t+k}$. Therefore, a claim to a *single* cash flow occurring at time $t+k$ is priced at:

$$P_t^{(k)} = E_t\left[\xi_{t,t+k} D_{t+k}\right].$$

Any asset with arbitrary maturity T (where T could be infinity) can be thought of as a "bundle" of individual claims with maturities $k = 1, 2, ..., T$, each paying the cash flow of the corresponding maturity, D_{t+k}. Under no arbitrage, the price of the bundle is the sum of the prices of the individual components:

$$P_t = \sum_{k=1}^{T} P_t^{(k)}. \tag{1}$$

A common alternative – equivalent – representation expresses prices in terms of expectations of future cash flows (*not* risk-adjusted cash flows) discounted at rates that incorporate a risk adjustment. For example, the price of the claim to the dividend occurring at time $t+k$ is represented as:

$$P_t^{(k)} = \frac{E_t\left[D_{t+k}\right]}{(1+\bar{r}_t^k)^k}, \tag{2}$$

where $\bar{r}_t^k$ is the per-period discount rate applied to the cash flow of maturity $t+k$. The discount rate $\bar{r}^k$ adjusts for both the timing of the dividend and the (maturity-specific) riskiness of the dividend.

For any asset, we can find an asset-specific discount rate $\bar{r}$ that makes the price equal to the present discounted value of the expected cash flows, all discounted at the same rate $\bar{r}$. For example, for an asset A with maturity T and price $P_{A,t}$, the corresponding discount rate $\bar{r}_{A,t}$ satisfies

$$P_{A,t} = \frac{E_t\left[D_{t+1}\right]}{1+\bar{r}_{A,t}} + \frac{E_t\left[D_{t+2}\right]}{(1+\bar{r}_{A,t})^2} + ... + \frac{E_t\left[D_{t+T}\right]}{(1+\bar{r}_{A,t})^T}. \tag{3}$$

Using Equations (1) and (2) we can also express $P_{A,t}$ as

$$P_{A,t} = \frac{E_t\left[D_{t+1}\right]}{1+\bar{r}_t^1} + \frac{E_t\left[D_{t+2}\right]}{(1+\bar{r}_t^2)^2} + ... + \frac{E_t\left[D_{t+T}\right]}{(1+\bar{r}_t^T)^T}, \tag{4}$$

where each individual cash flow is discounted at a maturity-specific rate $\bar{r}_t^k$. The two representations in Equations (3) and (4) are equivalent, implying that the per-period discount rate appropriate for a specific security, $\bar{r}_{A,t}$, can be thought of as a function of the maturity-specific discount rates $\bar{r}_t^k$ appropriate for each of the payments of that security.

This analysis emphasizes the importance of thinking about the maturity structure of any investment when deciding the appropriate discount rate. Consider for example a stream of 3 successive cashflows, D_t for $t = \{1, 2, 3\}$, that are expected to be \$10 at all future times, but with decreasing risk across maturity, such that the one-period-ahead cash flow should be discounted at 5% ($\bar{r}_t^1 = 5\%$), the two-period-ahead cash flow should be discounted at 3% ($\bar{r}_t^2 = 3\%$), and the three-period-ahead cash flow should be discounted at 1% ($\bar{r}_t^3 = 1\%$). Let investment A be a claim to all three cashflows. The price of this investment is:

$$P_{A,0} = \frac{10}{1.05} + \frac{10}{(1.03)^2} + \frac{10}{(1.01)^3} = \$28.66$$

The implied per-period discount rate is 2.33% since

$$\frac{10}{1.0233} + \frac{10}{(1.0233)^2} + \frac{10}{(1.0233)^3} = \$28.66 = P_{A,0}$$

Consider now an investment that is a claim only to the cashflow at time 3, i.e. a "long-run" investment. Clearly, the cash flow from this investment should be discounted at 1%, not at the 2.33% rate that is appropriate for the three-period "bundle" (investment A). This simple example illustrates how *even when restricting our attention to the same cash flows,* the appropriate discount rate for an investment is a function of the maturity structure of the investment. In general, to assess the appropriate discount rate to be applied to a specific investment, one needs to know the entire term structure of discount rates ($\bar{r}_t^1, \bar{r}_t^2 ...$), as well as the maturity structure of the investment's cash flows.

Finally, we remark that the term structure of discount rates $\bar{r}_t^1, \bar{r}_t^2$... reflects not only time discounting across horizons, but also the specific riskiness of cash flows at different horizons. In particular, it is easy to show that a cash flow D_{t+k} that is negatively correlated with the stochastic discount factor ξ is risky, and should be discounted at a rate $\bar{r}_t^k$ greater than the risk-free rate for that horizon. Conversely, a cash flow positively correlated with the stochastic discount factor is a hedge for priced risks, and should be discounted at a rate $\bar{r}_t^k$ lower than the risk-free rate for that horizon.

3 Leaseholds and Freeholds

In valuing investments that involve very long-run payoffs, it is crucial to obtain a reliable estimate of the discount rates $\bar{r}_t^k$ to be applied to cash flows arising far in the future (k of hundreds of years). GMS15, GMS16, and GMSW15 make progress on this question by studying a unique setup in the housing market that allows them to estimate these long-run discount rates. In particular, they exploit an institutional feature of housing markets in the U.K. and Singapore in which property ownership takes two forms: leasehold and freehold. A freehold corresponds to permanent ownership of a property, i.e. a claim to all future rents from the property. A leasehold is a grant of exclusive possession for a clearly defined and finite period of time (Burn et al., 2011). Common initial leasehold maturities are 99, 125, 150, 250 or 999 years. Unlike for commercial leases, in most cases the entire cost associated with a residential leasehold comes through the upfront purchase price. Leasehold properties are traded in liquid secondary markets, where the buyer purchases the remaining term of the lease. Once the leasehold expires, the ownership reverts back to the freeholder. GMS15 review in detail the rights and obligations of leaseholders and freeholders and find them comparable for the purpose of analyzing discount rates.

The empirical analysis in GMS15, GMS16, and GMSW15 is based on a comprehensive proprietary dataset of transaction-level administrative data on all residential housing sales in the U.K. and Singapore for the period 1995-2013. The data include the price paid for each property as well as various structural characteristics of the property. When the property is purchased under a leasehold the data include how many unexpired years remain on the lease at the time of sale.

We briefly review here the main characteristics of the U.K. data. GMS15 focus on 1.4 million transactions for flats (apartments) between 2004 and 2013. Figure I displays the distribution of remaining lease lengths for flats at the time of sale. There are many transactions with remaining lease lengths below 300 years and above 700 years; this variation makes it possible to trace out the term structure of leasehold discounts across maturities. About 3% of transactions are for freeholds, and 27% are for extremly long leaseholds (700 or more years remaining). The rest of the transactions are for shorter-maturity leaseholds.

Table I provides summary statistics for the main property characteristics in the U.K. sample. GMS15 group leasehold transactions by remaining maturity: 80-99 years, 100-124 years, 125-149 years, 150-300 years, greater than 700 years, and freeholds. As the table shows, the median flat in the U.K. is approximately 65 square meters large, with two bedrooms and one bathroom, and is located in a building that is 36 years old. The median price for a flat is £$123,000$. Property characteristics display some variation both between freeholds and leaseholds, and across leaseholds of different remaining lease length. The patterns, however, differ across characteristics. For example, freeholds tend to have more bedrooms but fewer bathrooms than leaseholds do, and tend to be larger. Shorter-maturity leaseholds and freeholds tend to be on older buildings than leaseholds of intermediate lease length. GMS15 show that this variation is drastically reduced once properties are compared only within the same three-digit postal code and conclude that properties held on leasehold contracts of varying maturity and on freehold contracts are overall very similar.

4 Interpreting Leaseholds' Discounts

To understand why the term structure of leasehold price discounts compared to freeholds is informative of discount rates, it is useful to think about the price difference between a hypothetical freehold and a leasehold on the same property.

Intuitively, since the underlying freeholder on a property that currently has a leasehold on it will receive the property back after the leasehold expires, the current difference in price between the freehold and the leasehold is the present value of receiving the freehold after the leasehold expires. More formally, the price difference between the freehold and the T-maturity leasehold is the current price of a claim to the freehold after the leasehold has expired (at T). One can compute this present value by applying the simple valuation formula: $P_t - P_t^T = \frac{E_t[P_T]}{R_{t,t+T}}$, where P_t is the price of a freehold at time t, P_t^T is the price of a leasehold with maturity T, and $R_{t,t+T}$ is the total discount rate appropriate for this claim. We obtain percentage leasehold discounts by dividing both sides by P_t:

$$Disc_t^T = -\frac{E_t[P_T]/P_t}{R_{t,t+T}} = -\frac{E_t[P_T]/P_t}{R^f_{t,t+T} + RP_{t,t+T}}, \tag{5}$$

where $R^f_{t,t+T}$ is the discount rate appropriate for a risk-free claim and $RP_{t,t+T}$ is the risk-premium adjustment due to the riskiness of rental income.[1]

This formula shows that leasehold discounts are related to two basic forces: the expected capital appreciation of the freehold (the numerator), and the discount factor (the denominator). The discounts are bigger the more households expect the price of the freehold to increase over the length of the leasehold. This is because the leaseholder does not benefit from these capital gains while the freeholder does. The discounts are also bigger the lower the discount factor, since this attaches higher present value to future rents.

5 A Reduced-Form Declining Discount Function

To make the general formula in Equation (5) operational it is convenient to specify a simple model of discounting and rent growth. We introduce here what turns out to be a convenient analytical functional form to match the term structure of leasehold discounts. Formally, in this reduced-form model, the total discount rate (that includes the risk-free rate and any risk premia) is a mix of hyperbolic and exponential discounting. It is beyond

[1] In the notation of Section 2 we write $R_{t,t+T} = (1 + \bar{r}_t^T)^T$ since this discount rate is the one appropriate for time-T cash flows.

the scope of this paper to provide a microfounded model explaining where such a discounting function might be coming from; here it is simply taken as exogenous and not to be interpreted as either a risk-free rate or a rate of time preference.

Moving for analytical simplicity to continuous time (so that the total discounting factor from 0 to t, $(1+\bar{r}_0^t)^{-t}$, can be written as $e^{-\bar{r}_0^t t}$) we assume that the discount function at time 0 for cash flows arising at time t is

$$e^{-\bar{r}_0^t t} = \frac{e^{-\rho t}}{1+\kappa t}, \tag{6}$$

where $\rho > 0$ controls the exponential component, and $\kappa > 0$ is the hyperbolic parameter. For $\kappa = 0$, the per-period discount rate is constant across maturities, $\bar{r}_0^t = \rho$ for all t, and the term structure of discount rates is flat. For $\rho = 0$, the term structure of discount rates has a hyperbolic shape: $\frac{1}{1+\kappa t}$. In general, the reduced form discount function in Equation (6) implies higher discount rates for short term than for long term cash flows. To illustrate this property, consider how the per-period discount rate differs across maturities: $\bar{r}_0^t = \rho + \frac{\ln(1+\kappa t)}{t}$, for the case $\rho > 0$ and $\kappa > 0$. Notice that we have $\lim_{t\downarrow 0} \bar{r}_0^t = \rho + \kappa$ and $\lim_{T\to\infty} \bar{r}_0^t = \rho$.[2] Therefore, per-period discount rates start at $\rho + \kappa$ and then decrease to ρ as maturity increases.

Note that the parameters ρ and κ should not be interpreted as deep primitives, but simply as convenient mathematical representations that allow us to capture the shape of the term structure of discount rates in a flexible way.[3] Since in this section we are not aiming to decompose the total discount rate into risk free and risk premium subcomponents, we assume for simplicity that rents grow at constant rate g.

Finally, note that in this setup, the T-maturity leasehold is valued at: $P_0^T = \int_0^T \frac{e^{-(\rho-g)s}}{1+\kappa s} D_0 ds$. Appendix A.1 derives analytic expressions for the resulting value, as well as for the value of the freehold.

[2]The first limit follows from an application of l'Hopital's rule.

[3]That is, we are not assuming that agents have either hyperbolic or exponential discounting, but simply that, whatever the underlying true model might be, the equilibrium discount rates can be approximately described by the assumed functional form.

6 Estimating Very Long-Run Discount Rates

GMS15 estimate the relative prices paid for leaseholds of varying maturity and freeholds using transaction data in the U.K. and Singapore. In each country, leaseholds are assigned to maturity buckets, based on the maturity remaining on the lease at the time of sale. GMS15 estimate the specification below:

$$log(P_{i,h,t}) = \alpha + \sum_{j=1}^{5} \beta_j \mathbf{1}_{\{T_{i,t} \in MaturityGroup_j\}} + \gamma Controls_{i,t} + \xi_h \times \psi_t + \epsilon_{i,h,t}$$

where $P_{i,h,t}$ is the price of a transaction i of a property in geographic area h at time t; $T_{i,t}$ is the maturity remaining on the leasehold at time of sale; the β_j coefficients capture the log price discount of leaseholds with maturity in bucket j of the MaturityGroup relative to otherwise similar freeholds, which are the excluded category in the regression; $Controls$ are various hedonic controls (like number of bathrooms and size of the property); and $\xi_h \times \psi_t$ indicates the interaction of postcode and time fixed effects.

Figure II reports the estimated β_j coefficients with their respective standard errors. Leaseholds with shorter maturities trade at greater price discounts to otherwise identical freeholds. Leaseholds with 80-99 years remaining trade at an approximately 16% discount to freeholds; the discount decreases to 10% for leaseholds with 100 to 124 years remaining, 8% for 125-149 years remaining, and 3% for 150-300 years remaining.

Interestingly, similar results hold in the case of Singapore, a country with a very different history and institutional environment. This suggests that the observed discount between freeholds and leaseholds is unlikely to be explained by country-specific institutional features. GMS15 further explores the possibility that the observed discounts may not capture maturity-specific discounting, but rather reflect confounding factors in these markets.[4]

GMS15 use the simplest model of finance, the Gordon growth model, to provide a

[4]GMS15 perform numerous robustness tests to rule out alternative hypotheses related to: systematic unobserved structural heterogeneity across different properties, differences in the liquidity of the properties, different clientele for the different ownership structures, and contractual restrictions in leasehold contracts.

back-of-the-envelope assessment of the discount rates implied by the leasehold/freehold price discounts. Assume that cash flows are discounted at a constant rate $\bar{r}$, and rents D_t grow over time at a constant rate g. This model is a special case of the one we introduced in Section 5 and is obtained by setting $\kappa = 0$. The price of a freehold is $\frac{D_t}{\bar{r}-g}$ and the price of a T-maturity leasehold is:

$$P_t^T = \frac{D_t}{\bar{r}-g}(1 - e^{-(\bar{r}-g)T}).$$

The percentage price difference between leaseholds and freeholds is:

$$Disc_t^T = -e^{-(\bar{r}-g)T}.$$

GMS15 calibrate g to the empirical real growth rate of rents (0.7% per year in the data), and show that a discount rate $\bar{r}$ of 2.6% per year fits the observed leasehold/freehold price discounts.

7 No-Bubble Condition

GMS16 show how the housing market setup with leaseholds and freeholds can be used to test empirically for the presence of bubbles. They focus on a particular type of bubble, the most prominent incarnation of which is a rational bubble, that satisfies the following properties:

$$P_t = \sum_{s=1}^{\infty} E_t[\xi_{t,t+s} D_{t+s}] + B_t, \qquad B_t \equiv \lim_{T\to\infty} E_t[\xi_{t,t+T} P_{t+T}], \tag{7}$$

where $\xi_{t,t+s} \equiv \prod_{j=0}^{s-1} \xi_{t+j,t+j+1}$, and

$$B_t = E_t[\xi_{t,t+1} B_{t+1}], \qquad \text{with} \quad B_0 > 0.$$

The term B_t is a bubble since it attaches positive present value to a claim that postpones indefinitely making any payments and has therefore zero fundamental value. Recall that for a finite maturity asset, like the T-maturity leasehold, we have:

$$P_t^T = \sum_{s=1}^{T} E_t[\xi_{t,t+s} D_{t+s}] \tag{8}$$

Subtracting (8) from (7) we obtain:[5]

$$P_t - P_t^T = E_t[\xi_{t,t+T} P_{t+T}]$$

Taking the limit as maturity of the leasehold goes to infinity, we obtain:

$$\lim_{T\to\infty} (P_t - P_t^T) = \lim_{T\to\infty} E_t[\xi_{t,t+T} P_{t+T}] = B_t, \tag{9}$$

where the last equality follows from the definition of the bubble in Equation (7).

The classic rational bubble has a long-standing tradition in the theoretical literature, with seminal papers by Samuelson (1958), Diamond (1965), Blanchard and Watson (1982), Tirole (1982, 1985), and Froot and Obstfeld (1991). It has since become the workhorse model of bubbles in macroeconomics (e.g., Caballero and Krishnamurthy, 2006; Arce and López-Salido, 2011; Martin, 2012; Martin and Ventura, 2014; Farhi and Tirole, 2012; Doblas-Madrid, 2012; Giglio and Severo, 2012; Gali, 2014; Galí and Gambetti, 2014; Caballero and Farhi, 2014).

Equation (9) is the basis of the empirical test strategy described in GMS16. They approximate the infinite limit by focusing on leaseholds of extremely long maturity: $T > 700$ years. GMS16 first show that rational bubbles were not present on average in their sample (U.K. and Singapore 1995-2013). They then also rule out such bubbles in subsamples across both time and geography. They focus on specific subsamples that were ex-ante more likely to have experienced a bubble.

Figure III reproduces some of the tests in GMS16. Panel A reports estimates for the U.K. of the 700-year leasehold discount to freehold estimated year by year between 1995

[5] See the Appendix in Giglio, Maggiori and Stroebel (2016) for full derivations.

and 2013. The difference in price is never statistically or economically significant despite substantial variation over time in house prices and house price to rent ratios. The remaining panels on Figure III focus on cross sectional analysis. GMS16 focus on 7,000 different geographic units, the MSOAs of England and Wales, and sort them in quintiles according to characteristics of the properties in each MSOA. They then estimate the 700-year leasehold discount to freehold in each quintile separately. Panel B displays the discounts when areas are sorted by their corresponding house price to income ratio. Panel C repeats the exercise sorting areas by the growth rate of the price to income ratio between 2004 and 2007. Panel D sorts areas by the average time on the market for properties on sale in each area. The rationale behind these cross sectional tests is that bubbles might be a priori more likely to be present in regions with higher house prices relative to income, with higher house price growth relative to income growth, and with lower time on market (more liquid or “hot" markets). GMS16 rule out the presence of rational bubbles across all these different sortings of the areas.

One notable feature of the test for bubbles in GMS16 is that it uses a purely cross-sectional (across assets) approach to identify bubbles, as opposed to a time-series approach. Many existing econometric tests for bubbles identify bubbly episodes by studying the time series behavior of prices compared to fundamentals (rents). These tests find it hard to distinguish between sharp and persistent movements in discount rates and the presence of a bubble. This occurs because both discount rate variation and the presence of a stochastic bubble induce sharp and persistent movements in prices compared to rents. GMS16’s test helps overcome this difficulty since it does not compare the *time-series* properties of prices to those of rents, but instead compares the prices of extremely-long leaseholds and freeholds at each point in time. Since the prices of both contracts are equally affected by variation in discount rates (the fundamental value of the two contracts is the same), but only the price of the freehold is affected by a bubble, the test can correctly account for (even sharp) movements in discount rates.

8 Climate Change and Long-Run Discount Rates

> *Any consideration of the costs of meeting climate objectives requires confronting one of the thorniest issues in all climate-change economics: how should we compare present and future costs and benefits? [...] A full appreciation of the economics of climate change cannot proceed without dealing with discounting.* (Nordhaus, 2013)

The literature on the economics of climate change, starting with Nordhaus (1973), has focused on the importance of discounting for evaluating the tradeoff between the immediate costs of climate change mitigation policy and its uncertain benefits that occur very far in the future.[6] An empirical literature has tried to infer the appropriate discount rates from the realized returns of traded assets such as private capital, equity, bonds, and real estate. For example, the dynamic integrated climate-economy (DICE) model of Nordhaus and Boyer (2000) and Nordhaus (2008) features a constant rate of discounting, calibrated to 4% to reflect the authors' preferred estimate of the average return to capital.[7]

GMSW15 first show theoretically that the appropriate inference from their estimates of discount rates for housing cashflows about the relevant discount rate for climate change abatement investments depends both on the horizon of the investment and on the relative risk properties of housing compared to investments in climate change mitigation. GMSW15 then provide new empirical evidence on the entire term structure of discount rates for housing cash flows as well as evidence of the riskiness of these cash flows.

As we reviewed in Section 2, the horizon of the investment matters because estimates of expected returns of assets capture only their average return, which might not reveal the appropriate discount rates for long-run claims if term structures of discount rates are not flat. For example, GMSW15 estimate that the average returns to residential housing are in the range of 6-8%. However, the relevant estimates to evaluate long-run housing claims are

[6] See also: Arrow et al. (1996); Weitzman (1998); Groom et al. (2005); Gollier (2006); Nordhaus (2007); Weitzman (2007); Pindyck (2013); Greenstone et al. (2013).

[7] Stern (2007) argues for 0% discount rate on ethical grounds that require the present generation to not discount the welfare of future generations. Nordhaus (2007) points out that a 0% discount rate cannot be reconciled with economic theory because it would imply an enormous burden on the current generation by attaching infinite values to many investments that are routinely available in private markets at finite prices.

the discount rates for cash-flows very far into the future. GMSW15 found such discount rates to be much lower than those implied by average returns and of the order of 2.6% for 100-year claims on housing. They conclude that the term structure of discount rates for real estate cash flows must be downward sloping. Figure IV shows that the reduced form declining discount function we introduced in Section 5 is capable of fitting both the low long-run discount rates and the high average rate of return (in the calibrated version, the expected return of a freehold is about 6%).[8]

GMSW15 then turn to the riskiness of housing cash flows and show that housing across countries and time periods is a risky asset: it has low returns in periods of financial crises, rare disasters, and wars. Finally, in ongoing work GMSW15 combine these new empirical estimates with structural models of risk premia to provide guidance to the climate change discounting literature; discount rates at or below 2% are likely to be appropriate for climate change abatement investments, when such investments mitigate the probability or the effects of climate-change-induced aggregate risk.

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[8] We calibrate $\rho = 0.0142$ and $\kappa = 0.12$ at the yearly frequency.

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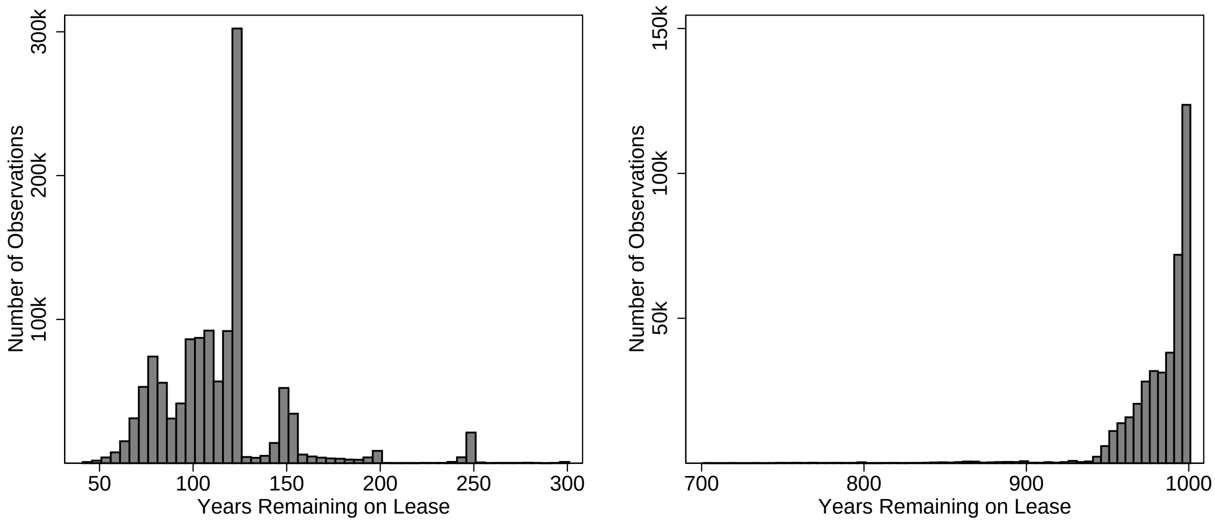
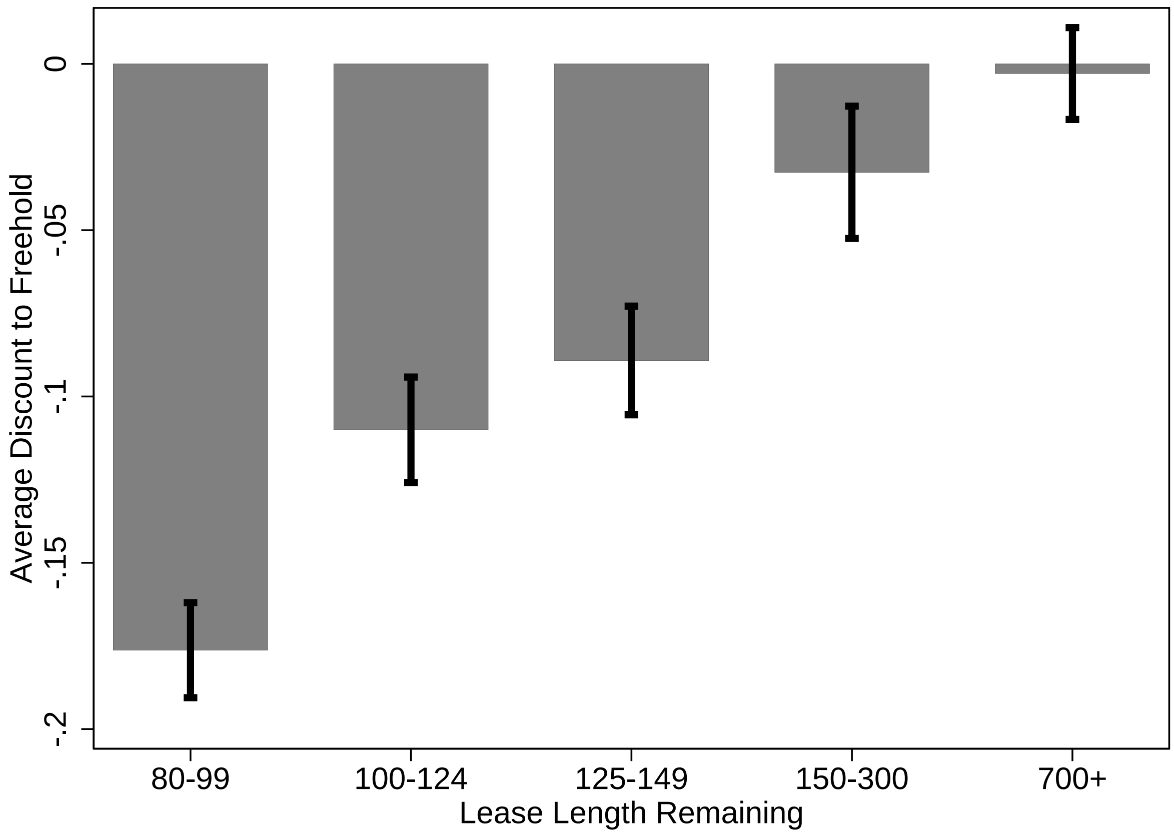
Figure I: Distribution of Observed Maturities



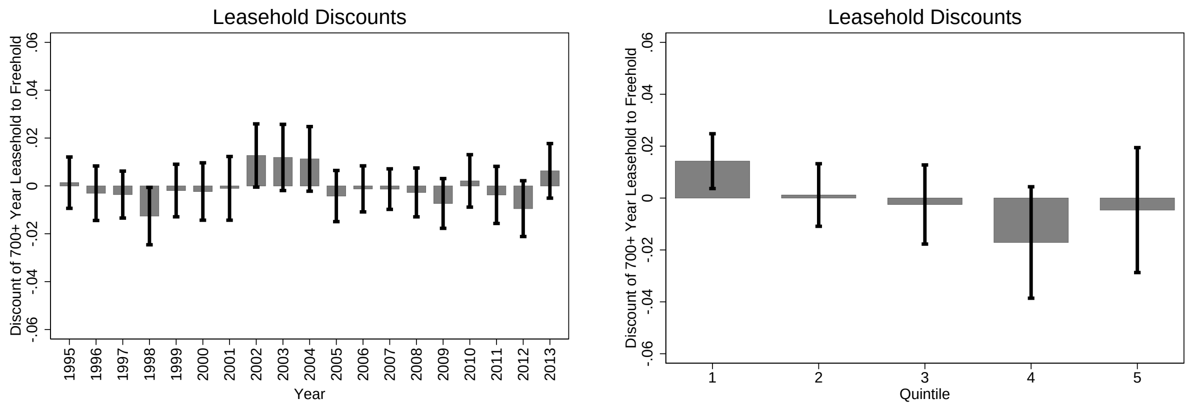
Note: Source: GMS15. The figure shows the distribution of remaining lease length at the point of sale for flats in the U.K. sample. See original reference for further details on the data.

Figure II: Leasehold Price Discounts by Remaining Lease Length



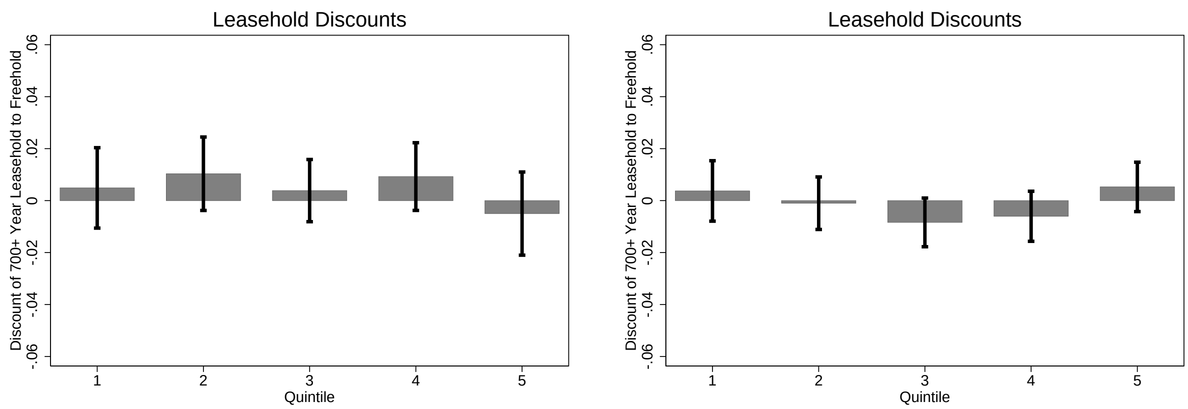
Note: Source: GMS15. The figure plots log-price difference between leaseholds of varying remaining maturity and freeholds for flats sold in the U.K. between 2004 and 2013. The vertical axis is expressed in log-points, the horizontal axis shows leasehold discounts depending on the remaining term of the lease. The bars indicate the 95% confidence interval of the estimate. See original reference for further details on data and estimation.

Figure III: Time-Series and Cross-Section of Bubble Claim



(A) By year

(B) By price-income ratio (2004)

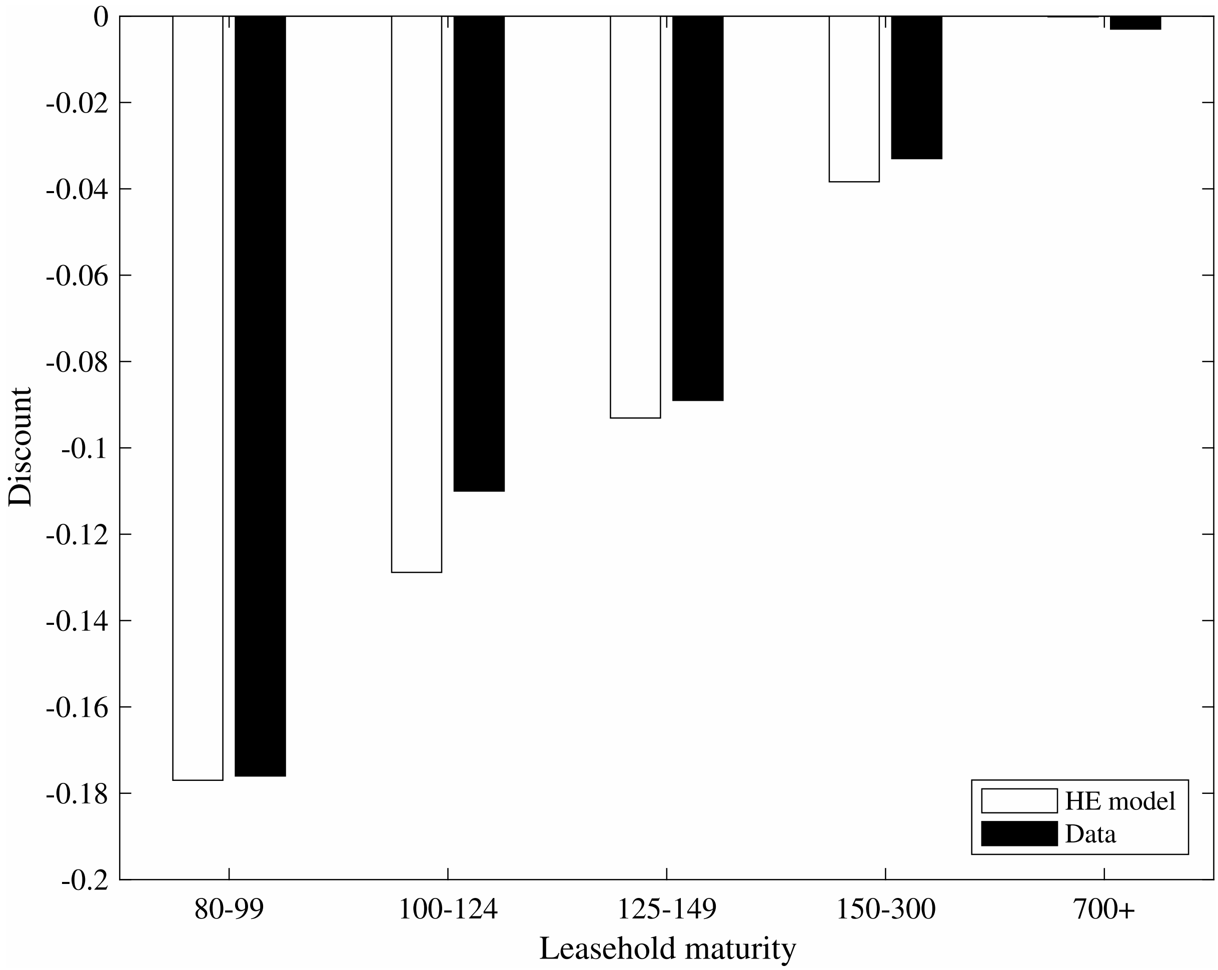


(C) By price-income ratio growth (2004-7)

(D) By time-on-market

Note: Source: GMS16. The figure reports estimates of the discount between 700+ year leaseholds and freeholds from regression 6, dividing the U.K. sample along time-series and cross-sectional dimensions. Panel A shows the coefficients of the 700+ leasehold discount year by year. Panels B through D report the coefficients of the 700+ leasehold discount, splitting Middle Layer Super Output Areas by quintiles of measures of the potential for a bubble: the price-income ratio in 2004 (Panel B), the growth of the price-income ratio between 2004 and 2007 (Panel C), and the time-on-market (Panel D). The bars indicate the 95% confidence interval of the estimate using standard errors clustered at the 3-digit postcode level. See original reference for further details on data and estimation.

Figure IV: Discounts generated by the Hyperbolic-Exponential model



Note: The figure shows the log price discounts for leaseholds observed in U.K. together with the discounts implied by a parameterizations of the hyperbolic-exponential discounting model. The model generates expected returns for freeholds of about 6% per year.

Table I: Summary Statistics

Variable	Lease Length	Mean	Stdev	PERCENTILE p1	p5	p25	p50	p75	p95	p99
Price (£'000)	80-99	121.1	125.7	18	29	57	91	149	290	545
	100-124	155.0	145.0	21	36	80	130	190	350	610
	125-149	177.6	183.6	25	52	103	145	205	380	750
	150-300	175.2	146.7	26	46	103	146	210	385	650
	700+	176.0	242.9	20	33	75	125	202	460	950
	Freehold	140.9	191.6	15	27	59	105	163	359	780
	TOTAL	155.6	178.1	20	34	73	123	185	371	712
Bedrooms	80-99	1.66	0.65	1	1	1	2	2	3	3
	100-124	1.79	0.66	1	1	1	2	2	3	4
	125-149	1.83	0.60	1	1	1	2	2	3	4
	150-300	1.80	0.58	1	1	1	2	2	3	3
	700+	1.84	0.65	1	1	1	2	2	3	4
	Freehold	2.33	0.98	1	1	2	2	3	4	5
	TOTAL	1.79	0.66	1	1	1	2	2	3	4
Bathrooms	80-99	1.08	0.29	1	1	1	1	1	2	2
	100-124	1.17	0.40	1	1	1	1	1	2	2
	125-149	1.29	0.50	1	1	1	1	2	2	3
	150-300	1.27	0.46	1	1	1	1	2	2	2
	700+	1.21	0.44	1	1	1	1	1	2	3
	Freehold	1.17	0.47	1	1	1	1	1	2	3
	TOTAL	1.17	.40	1	1	1	1	1	2	2
Size (m^2)	80-99	66.3	48.2	29	35	49	60	73	103	161
	100-124	71.9	55.0	30	40	54	66	79	108	180
	125-149	74.0	52.4	33	43	57	67	79	115	200
	150-300	71.1	42.9	31	41	55	66	78	111	162
	700+	75.6	62.7	30	39	54	67	82	127	212
	Freehold	94.0	45.0	42	49	71	96	99	152	237
	TOTAL	72.2	54.9	30	39	53	65	80	115	190
Age (years)	80-99	60.3	48.4	0	3	15	56	101	127	165
	100-124	44.8	44.1	0	0	10	35	67	121	158
	125-149	37.4	49.4	0	0	1	9	69	123	160
	150-300	39.4	48.9	0	0	1	21	73	123	162
	700+	52.2	60.0	0	0	10	35	97	144	205
	Freehold	61.2	56.7	0	2	19	45	100	146	253
	TOTAL	50.3	48.7	0	0	10	36	95	128	179

Note: Source: GMS15. The table shows summary statistics for the main hedonic variables for the sample of U.K. flats. For each characteristic, the table reports the statistics separately for different buckets of remaining lease length, as well as for the pooled sample. See original reference for further details on the data.

A.1 Appendix: Details on Hyperbolic-Exponential Discounting

We include here details for the derivations in Section 5 of the paper. First, let us focus on a model where the discount rate by maturity is purely hyperbolic. In continuous time, the hyperbolic discount function is simply $\frac{1}{1+\kappa s}$ where $\kappa > 0$ is the hyperbolic parameter. To gather intuition, assume that rents were constant at D. Let us value the T-maturity lease contract. For simplicity consider valuation at $t = 0$.

$$P_0^T = \int_0^T \frac{1}{1+\kappa\, s} D ds = D \frac{\ln(1+\kappa\, T)}{\kappa}.$$

The obvious problem with this model for the term structure of discount rates when applied to longer term assets is that the valuation of claims diverges (even without dividend growth) as the horizon T increases ($T \to \infty$).

In the paper, therefore, we augmented the hyperbolic discount function to include an exponential term: $\frac{e^{-\rho s}}{1+\kappa s}$, where $\rho > 0$ is the exponential component of the discount rate. This mixed hyperbolic-exponential form of discounting tends to behave like hyperbolic discounting in the short run and like exponential discounting in the long run. Since the long-run discount rate approaches ρ, finite prices for long-run securities in the presence of cash-flow growth g are guaranteed by $\rho > g$. The T-maturity leasehold is valued at:

$$P_0^T = \int_0^T \frac{e^{-(\rho-g)s}}{1+\kappa s} D_0 ds = D_0 \frac{e^{\frac{\rho-g}{\kappa}} \left(Ei\left(\frac{(T\kappa+1)(g-\rho)}{\kappa}\right) - Ei\left(\frac{g-\rho}{\kappa}\right)\right)}{\kappa},$$

where $Ei(x)$ is the Exponential Integral function defined as:

$$Ei(x) \equiv -\int_{-x}^{\infty} \frac{e^{-t}}{t} dt.$$

The freehold is correspondingly valued at:

$$P_0 = D_0 \frac{e^{\frac{\rho-g}{\kappa}} \Gamma\left(0, \frac{\rho-g}{\kappa}\right)}{\kappa},$$

where $\Gamma(x)$ is the Upper Incomplete Gamma Function defined as:[1]

$$\Gamma(0, x) \equiv \int_{x}^{\infty} \frac{e^{-t}}{t} dt.$$

The leasehold-freehold discount is now:

$$Disc_0^T = \frac{Ei\left(\frac{(T\kappa+1)(g-\rho)}{\kappa}\right) - Ei\left(\frac{g-\rho}{\kappa}\right)}{\Gamma\left(0, \frac{\rho-g}{\kappa}\right)} - 1.$$

[1] Notice $\Gamma(0, x) = -Ei(-x)$.

The per-period equivalent constant discount rate $\bar{r}_0^T$ for any horizon T solves $e^{-\bar{r}_0^T T} = (R_{0,T})^{-1} = \frac{e^{-\rho T}}{1+\kappa T}$, and is hence obtained via the formula:

$$\bar{r}_0^T = \rho + \frac{\ln(1+\kappa T)}{T}.$$

This is the formula reported in the main text. Notice that we also have $\lim_{T\downarrow 0} \bar{r}_0^T = \rho + \kappa$ and $\lim_{T\to\infty} \bar{r}_0^T = \rho$.[2] So that total discount rates start at $\rho + \kappa$ and then decay over the horizon to ρ.

Similarly, marginal discount rates $r(s)$ can be derived by defining the discount function $F_{0,T} = exp\left(-\int_0^T r(s)ds\right)$. Then an application of Leibniz's rule for differentiation under the integral sign yields: $\dot{F_{0,T}} = -r(T)F_{0,T}$, where $\dot{F_{0,T}}$ is the time derivative of function $F_{0,T}$. Hence, we have the result hat $r(T) = -\frac{\dot{F_{0,T}}}{F_{0,T}}$. Finally, applying this formula to the exponential-hyperbolic discount function, $F_{0,T} = \frac{e^{-\rho T}}{1+\kappa T}$, one obtains the result:

$$r(T) = -\frac{\dot{F_{0,T}}}{F_{0,T}} = \rho + \frac{\kappa}{1+\kappa t}.$$

Marginal discount rates are therefore monotonically decreasing from $\rho + \kappa$ to ρ.

We next derive the expected, instantaneous returns to the freehold, under the assumption that the term structure of discount rates is constant over time (hence, this instantaneous return will also be the expected return). Before deriving the expression for the hyperbolic-exponential model, we report the derivation for the simple Gordon growth model where all cash flows are discounted at the same rate r (flat term structure of discount rates). The instantaneous return on the freehold is given by:

$$\frac{dP_t + D_t dt}{P_t}.$$

In the Gordon growth environment with a flat term structure of discount rates, capital gains are $\frac{dP_t}{P_t} = gdt$. This can be derived recalling that $P_t = \frac{D_t}{r-g} = \frac{D_0 e^{gt}}{r-g}$ and taking the time derivative. The rental yield is $\frac{D_t}{P_t} = r - g$. We conclude that total returns on the freehold in the Gordon growth model are:

$$\frac{dP_t + D_t dt}{P_t} = gdt + (r-g)dt = rdt.$$

We now derive the formula for expected returns to the freehold in our hyperbolic-exponential model by analogy with the Gordon growth model derivation above. The capital gains in our hyperbolic-exponential model are $\frac{dP_t}{P_t} = gdt$. This can be derived by recalling that $P_t = D_t \frac{e^{\frac{\rho-g}{\kappa}}\Gamma\left(0,\frac{\rho-g}{\kappa}\right)}{\kappa} = D_0 e^{gt} \frac{e^{\frac{\rho-g}{\kappa}}\Gamma\left(0,\frac{\rho-g}{\kappa}\right)}{\kappa}$, and taking the time derivative. The rental yield is $\frac{D_t}{P_t} = \frac{\kappa}{e^{\frac{\rho-g}{\kappa}}\Gamma\left(0,\frac{\rho-g}{\kappa}\right)}$. We conclude that total returns on the freehold in the hyperbolic-exponential model are:

$$\frac{dP_t + D_t dt}{P_t} = gdt + \frac{\kappa}{e^{\frac{\rho-g}{\kappa}}\Gamma\left(0,\frac{\rho-g}{\kappa}\right)}dt.$$

[2]The first limit follows from an application of l'Hopital's rule.

If $\kappa = 0$ then the return to the freehold is simply ρ, and we are back to the exponential discounting model. An increase in κ for a given ρ has the following comparative statics: the returns to the freehold increase, short term discount rates increase, long-term discount rates are unchanged, and leasehold discounts $(Disc)$ increase in absolute value. These dynamics are precisely what allow the reduced-form hyperbolic-exponential model to reconcile the long-run valuation pattern.