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Intangible capital, relative asset shortages and bubbles

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ABSTRACT

Purely technological factors can be a fundamental force behind the emergence of asset price bubbles in developed economies. We analyze an economy in which the production technology utilizes both physical and intangible capital, where the latter cannot be used as collateral for borrowing. Technological change, in the form of increased importance of intangible capital in production, sharpens the borrowing constraints of entrepreneurs, leading to a scarcity of high-yield assets relative to low-yield ones. This can create the conditions for asset bubbles. Additionally, due to the financial frictions, standard dynamic efficiency tests are not valid, and bubbles are not Pareto improving.

1. Introduction

There is enormous fascination among economists by the topic of asset price bubbles, both at the theoretical and at the practical level. This is attested by the large volume of studies on the subject, recently surveyed in Brunnermeier (2009). Yet, the general knowledge about the causes and consequences of these important phenomena is still limited. The recent boom and bust of the housing sector in the United States and its widespread effects worldwide warn us about the importance of understanding why bubbles emerge, how they gain momentum and finally why they burst. An important open question in the literature is whether there are structural conditions in the global economy stimulating the creation of bubbles and, if that is the case, what governments should do.

The central contribution of our paper is to highlight that the dynamics of capital accumulation in *developed* countries might have induced a shortage of high-yield financial assets and a surplus of low-yield securities in capital markets, creating the conditions for the existence of asset price bubbles. Our theory builds on the fact that, in the recent decades, the importance of intangible capital – research and development (R&D), information technology, advertising, firm-brand, etc. – has increased relative to physical capital as an input in the production processes of advanced economies. By its very nature, intangible capital is not as good a source of collateral as tangibles. Other things equal, a higher dependence on intangibles distorts the economy's ability to issue financial assets in order to borrow funds and invest, affecting the mix of stores of value available to investors. The resulting scarcity of high-yield securities is the key mechanism to sustain bubbly equilibria.

More specifically, the paper analyzes an overlapping generations (OLG) economy in which entrepreneurs raise funds to invest both in physical and intangible capital. Due to a simple moral hazard problem, borrowing requires collateral, which can only be provided by physical capital. As the importance of intangibles in production grows relative to tangibles, entrepreneurs face a harder time supplying assets in order to raise funds to invest in intangible capital. Since in equilibrium savings are to be invested, they end up financing an over-accumulation of physical structures. In turn, the high

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stock of physical capital drives its marginal productivity down, reducing the interest rate faced by lenders. Under those circumstances, rational bubbles can be sustained in equilibrium.[1]

From a methodological perspective, the present paper is closely related to a recent strand of the economics literature suggesting that speculative bubbles and other important phenomena result from a chronic problem of asset shortage in the world economy. According to Caballero (2006): "Asset supply is having a hard time keeping up with the global demand for stores of value and collateral by households, corporations, governments, insurance companies, and financial intermediaries more broadly". A simple consequence of this mismatch between global supply and demand is price appreciation, inducing speculation and the frequent occurrence of bubbles. If this is actually the case, one wonders why the world economy would be unable to increase the supply of assets at the same pace as demand for them rises.

Caballero et al. (2008a, 2008b) provide an answer. The authors argue that heterogeneity in the degree of financial development across countries coupled with the fast growth and high savings rates in developing economies are at the root of the matter. These fast growing and high savings areas have accumulated substantial wealth in the recent decades, increasing their demand for stores of value. Because financial markets and legal institutions in general are less developed in these regions, the issuance of new assets to accommodate the increased demand has not grown fast enough. In stylized theoretical contexts, this phenomenon results in the existence of equilibria featuring rational bubbles.[2]

We shift the focus of the analysis in two important dimensions. First, by emphasizing that the crucial aspect of the problem is not an *absolute* shortage of stores of value, but instead a *relative* scarcity of high-yield securities and an abundance of low-yield ones. Second, by claiming that, at least in part, the source of this imbalance is found in the developed nations whose production processes rely extensively on intangible capital. This, in a certain sense, addresses an important criticism to the asset shortage literature, which falls short of explaining why the highly developed financial markets of advanced economies would be unable to properly accommodate the inflow of savings from emerging countries. Our model shows that advanced economies can in fact absorb the excess savings from abroad. The central issue is that they can only do so by over-investing in relatively unproductive assets, since frictions prevent an efficient allocation of funds.

Another difference of our model relative to most previous papers is that it considers a production technology that displays declining marginal productivity of factors. The existing models of asset shortage have relied extensively on linear technologies with marginal productivities that are forced to be higher than the equilibrium interest rate lenders face. Because of asymmetric information, any additional funds can only partly be used for productive investment. This creates a gap between the supply and demand for stores of value, providing the conditions for the emergence of bubbles. In the present paper, on the contrary, lenders can always allocate their funds to real investments. However, after a certain point, moral hazard constraints become relevant and savings can only be channeled to the accumulation of physical capital, driving down its marginal productivity and the rate of interest creditors obtain.

The downward pressure on interest rates is the key mechanism linking technologic al advances to bubbles in our paper. As pointed out by Tirole (1985) and Santos and Woodford (1997), the low interest rate is a necessary condition for equilibria featuring rational bubbles when information is symmetric. In the OLG model of Tirole, for example, rational bubbles require the equilibrium interest rate to be below the growth rate of the economy in steady-state. This has been traditionally interpreted as a sign of dynamic inefficiency resulting from the over-accumulation of capital.

The close connection between rational bubbles and dynamic inefficiency has been a source of criticism to models of rational bubbles. For instance, the classic paper of Abel et al. (1989) – henceforth AMSZ – provides evidence that actual economies are not inefficient. More specifically, AMSZ compared the inflow versus the outflow of funds from the corporate sectors of OECD economies and found that they consistently generate more resources than what they absorb, a clear sign of efficiency. Moreover, in the context of their paper, AMSZ prove that their empirical test is equivalent to the interest rate criterion in Tirole's economy. However, in the presence of financial frictions, the link between interest rates and dynamic efficiency is severed, as shown by Woodford (1990), Farhi and Tirole (forthcoming), and Martin and Ventura (forthcoming). These papers demonstrate that economies which are efficient according to the AMSZ criterion can sustain rational bubbles in equilibrium.

Our paper builds extensively on this result, combining it with the role of intangible capital. In particular, the paper proves that, when intangible capital has little importance, rational bubbles cannot exist unless the bubbleless economy fails the AMSZ empirical benchmark. As intangibles become more relevant, on the other hand, rational bubbles can always be sustained in equilibrium. Hence, technological evolution moves the economy into a bubbly region and yet allows it to be productive, in accordance with the existing evidence. The reason why the model passes the AMSZ test despite the low interest rate is that the high stock of physical capital makes intangibles highly productive.

The paper also contributes to the literature on dynamic efficiency, complementing the previous papers and formally proving that, in the presence of financial frictions, not only the interest rate test but also the AMSZ test of dynamic efficiency is not appropriate. Despite our stylized framework, this result is much more general. Financial frictions create

[1] Our focus on rational bubbles is not meant to fully capture the pattern of speculation in financial assets observed in reality. The analysis here is confined to fully rational environments because they impose more discipline on the researcher: intuitively, the conditions for rational bubbles are more stringent than those for behavioral ones. This, in a certain sense, reinforces the main findings in the paper. If technological evolution and the consequent relative asset shortage are capable of inducing bubbles even in fully rational environments, it is not hard to imagine that they can do even more once investors are allowed to make systematic mistakes.

[2] Mendoza et al. (2007) present a model with similar predictions.

heterogeneity in the investment opportunities that different individuals face over their lifetime, and introduce an additional margin to improve allocations. Building on this intuition, we demonstrate that the AMSZ test is not equivalent to dynamic efficiency, even when potential reallocations are required to obey the existing financial constraints. In spite of that, rational asset price bubbles are not Pareto improving because they cannot replicate the type of reallocation proposed.

Our paper is part of a broader literature on the interaction between bubbles and economic growth. Grossman and Yanagawa (1993), King and Ferguson (1993), and Olivier (2000), for example, analyze models of endogenous growth due to externalities in the accumulation of capital. In these papers, production externalities create a wedge between the private and social rates of return on investments, allowing for bubbly equilibria. Our work assumes no externalities but instead focuses on the role of financial frictions as the source of wedges that permit the co-existence of rational bubbles and efficiency in the AMSZ sense.

Similar to the present paper, other authors have studied the role of frictions in capital markets and their interaction with bubbles as well. Kocherlakota (1992, 2008), Woodford (1990), Farhi and Tirole (forthcoming) and Martin and Ventura (forthcoming) demonstrate how bubbles can arise in the presence of different types of financial constraints, either with infinitely lived agents or in OLG economies. Contrary to ours, these papers are not based on a mix of inputs with different pledgeability. Kiyotaki and Moore (1997) and Lorenzoni (2008) study the macroeconomic implications of financial frictions, through the fluctuation in the value of collateral and fire-sale externalities. These papers focus on the additional macroeconomic volatility stemming from incomplete risk sharing, whereas ours concentrates on the effect of frictions on capital accumulation and bubbles.

Several studies have analyzed rational bubbles in the context of asymmetric information, where the existence of the bubble is not common knowledge (Allen et al., 1993; Conlon, 2004) or where speculators are more informed than their investors (Allen and Gorton, 1993; Allen and Gale, 2000; Barlevy, 2008). Harrison and Kreps (1978) and Scheinkman and Xiong (2003), on the other hand, focus on the interaction between heterogeneous beliefs and short sale constraints, which increase the value of assets beyond fundamentals thanks to the option to sell them in the future to agents with different beliefs. Our model is distinct from those since both the information structure and beliefs are symmetric.

The rest of the paper is structured as follows. Section 2 discusses the evidence on intangible capital and the role of collateral constraints. Section 3 presents the model, while Sections 4 and 5 characterize its equilibria. Section 6 builds on the technical results derived before to expose our connection between technological progress and bubbles. Section 7 discusses the subject of dynamic efficiency, and Section 8 concludes. All proofs of propositions as well as extensions of the basic model are collected in the Technical appendix.[3]

2. Evidence on intangible capital

There are two central assumptions in the model. First, the importance of intangible capital relative to physical capital in production has increased over time in developed countries. Second, physical capital is a better source of collateral compared to intangibles, determining entrepreneurs' ability to borrow funds and invest. This section discusses several papers that present evidence supporting these two hypotheses.[4]

2.1. The increased importance of intangible capital

A building block in the model to be presented later is the technology entrepreneurs use to produce the single final good:

$$y = Ak^{\alpha} z^{\beta} l^{1-\alpha-\beta} \tag{1}$$

where A is a measure of productivity, k represents physical capital, z represents intangible capital and l stands for labor. Several papers have employed similar versions of this specification to analyze the effects of intangible capital.

McGrattan and Prescott (2007) adopt an identical technology to evaluate the ability of the otherwise conventional neoclassical growth model to account for the business cycle in the United States. economy during the 1990s. The authors show that including intangibles significantly improves the performance of the calibrated model. Hall (2000) introduces the concept of E-capital as an input for final production, which for many purposes is equivalent to our notion of intangibles. Relying on a non-parametric approach to calculate the stock of E-capital, Hall claims it might be an important common element behind events in the United States stock and labor markets in the 1990s.

In competitive equilibria the exponents of the Cobb–Douglas production function represent the share of total income that accrues to each factor. This is a measure of the importance of each factor for production, and it is the one used in the paper. Hence, the notion that the importance of intangible capital has increased over time relative to physical capital translates, in our model, in an increase of β relative to α. Note that this assumption is not equivalent to saying that the

[3] The online appendix is available on Elsevier's Science Direct website.

[4] In the next section several additional assumptions are introduced in order to obtain closed-form expressions and to illustrate the main theoretical results of the paper. These assumptions are made for the sake of simplicity and are by no means crucial to obtain the intuition behind our findings. What is central is the increased importance of intangibles relative to tangibles and the interaction between the low collateralizability of intangibles and the borrowing constraints.

Table 1
Shares of income to factor, non-farm business sector, 2000–2003, including and excluding intangible capital (from Corrado et al., 2009, Table 3).

	Conventional, without intangibles	Including intangibles
Shares of total income		
Labor	70.4	60.0
Tangible capital	29.6	25.0
Intangible capital		15.0

stock of z has increased more than the stock of k. Such a movement could result from differences in initial conditions and general equilibrium effects, even without any change in the structure of the production function. The statement that β has increased relative to α is essentially an assumption about a change in the nature of the production process in developed countries. There is strong evidence suggesting this is actually the case.

Corrado et al. (2009) measure the share of output earned by the owners of intangibles in the United States. Their definition of investment in intangible capital includes expenses on firm brand, firm-specific resources, scientific and non-scientific R&D and computerized information. Using data on national accounts and estimates presented in Corrado et al. (2005), the authors show that investment on intangibles has increased substantially in the recent decades, much more so than investment in physical capital. More importantly, their calculations indicate that the fraction of non-farm business output in the United States accruing to intangible capital has increased from 9.4% on average in the 1973–1995 period to approximately 14% in the 1995–2003 period. For the years 2000–2003, the share of income earned by the owners of intangible capital reaches 15%, while the owners of physical capital obtain 25% and the remaining 60% is absorbed by labor. Since there is no evidence that the share of income earned by physical capital has increased, when translated to our model, these results confirm that the ratio β/α has increased in the United States in the last decades. At the same time, the data also point to a general shift of production from labor to capital (an increase in $\alpha+\beta$). Table 1 summarizes this information.

Belhocine (2009) reproduces Corrado et al. (2009) and finds qualitatively similar evidence for Canada, where estimated investment in intangibles has become as large as the investment in physical capital. Fukao et al. (2009) and Marrano and Haskel (2007) present similar findings for Japan and the United Kingdom respectively.

2.2. *Intangible capital, collateral and financing constraints*

Collateral is an important instrument for borrowers to raise funds in imperfect capital markets. Moral hazard and adverse selection reduce lenders' willingness to allocate capital to borrowers whose characteristics or actions are not completely observable. Collateral plays an important role in overcoming this failure sice it allows lenders to seize assets in the event of default. Moreover, it increases borrowers' incentives to be diligent and perform well.

As emphasized by Shleifer and Vishny (1992), what turns an asset into good collateral is its redeployability in the contingency of liquidation. In other words, good collateral is an asset which can be costlessly seized and sold in the market for a price similar to its value in the first best use. There is ample empirical evidence associating redeployability of collateral and access to financing.[5]

Intangible capital is, in general, a worse source of collateral compared to physical capital. First and foremost, intangible capital is, by its very nature, hard if not impossible to seize. Furthermore, it is fundamentally intrinsic to the firm or business that produced it. Even if, in some cases, lenders can obtain control over the intangibles of defaulting borrowers, the redeployability of these assets tends to be small since their value in alternative uses is reduced by the lack of specific knowledge or competences. Obviously, different sorts of physical capital suffer, to some extent, from the same problems. However, plants, equipment and machines are generally accepted by financial intermediaries as collateral for lending.

The second fundamental assumption in our paper builds on this reasoning to argue that, other things equal, the more firms rely on intangible capital the more the financing constraints bind. Several studies have documented this fact. Aghion et al. (forthcoming) show that R&D investment correlates with financing constraints in a large sample of French firms. Carpenter and Petersen (2002) find that small high-tech U.S. firms suffer more from financing constraints. Canepa and Stoneman (2008) present similar evidence for firms in the United Kingdom.

3. Model

The economy is represented by an OLG model with one final good (y) and three inputs: labor (l), physical capital (k), and intangible capital (z). At every period t, a new unit-measure generation is born with an endowment of 1 unit of time and no endowment of goods. Each generation is composed by a fraction π of entrepreneurs and a fraction $1-\pi$ of households. Individuals live for three periods: young, middle age, and old. They only consume when old, and their utility is linear in consumption: $U_t(c_t(t),c_t(t+1),c_t(t+2))=c_t(t+2)$, where $c_t(s)$ is consumption of generation t at time s. All individuals will

[5] See for example Benmelech et al. (2005) and Benmelech (2009).

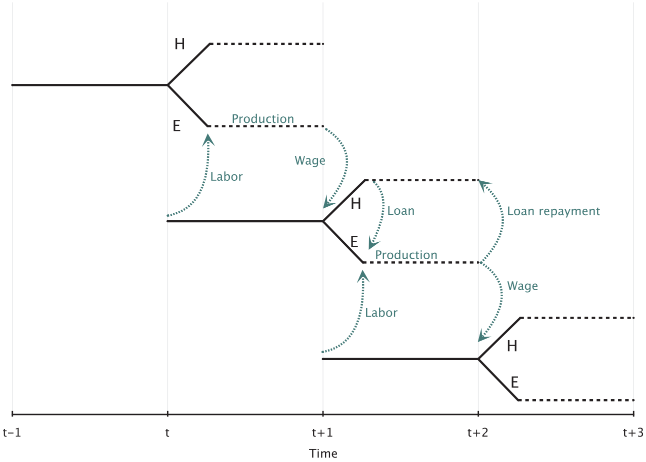

Fig. 1. The overlapping-generation structure of the economy. Each generation lives for three periods, young, middle-aged, and old. When young, they provide labor to previous generations, receiving wages in the next period. When middle-aged, households lend their savings to entrepreneurs of the same generation.

use at best production and investment opportunities during their lifetime to convert the initial endowment of time into end-of-life consumption. For simplicity, there is no uncertainty in the economy.

Fig. 1 illustrates the model by describing the evolution of the generation born at time t. When young, both households and entrepreneurs supply labor inelastically to the existing firms run by the middle-aged entrepreneurs of generation $t-1$ in a competitive labor market. Labor services are paid only at the beginning of period $t+1$, at a wage rate $w(t+1)$. The labor supply of the young will represent the aggregate labor supply of the economy in each period.

At the beginning of period $t+1$, households have no option but to lend their accumulated labor income at a gross rate $R(t+2)$ – to be paid at date $t+2$ – to the contemporaneous entrepreneurs in order to transfer funds to the last period of life. Hence, at date $t+2$, a household of generation t consumes her total wealth, represented by $w(t+1)R(t+2)$, and then dies.

Entrepreneurs, on the other hand, have different investment opportunities. They use their own funds and borrow additional resources from the contemporaneous households to accumulate physical and intangible capital. They combine accumulated capital with labor services from the newborn of generation $t+1$ to produce final goods. Production by generation t takes place between periods $t+1$ and $t+2$ and generates output of the final good at the beginning of time $t+2$, denoted $y_t(t+2)$. Note that $y_t(t+2)$ refers to the output produced by a single entrepreneur.

Finally, at date $t+2$, the total output produced by generation t is distributed among the owners of the factors. Part of it goes to the old households of generation t who lent funds to the entrepreneurs. Another part remunerates the labor services provided by generation $t+1$. The remaining output is retained by the old entrepreneurs of generation t, who consume and die.

Each middle-aged entrepreneur of generation t has access to a Cobb–Douglas production technology that combines physical capital k, intangible capital z and labor l as follows:

$$y_t(t+2) = A(t+2)k_t(t+1)^{\alpha} z_t(t+1)^{\beta} l_t(t+1)^{1-\alpha-\beta} \tag{2}$$

A is a productivity parameter that grows at the constant rate n and $l_t(t+1)$ represents the individual demand for labor in period $t+1$. At the beginning of $t+1$, a typical middle-aged entrepreneur uses her own funds $w(t+1)$ and borrows additional resources $d_t(t+1)$ to accumulate $z_t(t+1)$ and $k_t(t+1)$. This is the first stage of production. In a second and final stage, she hires labor from the newborn of generation $t+1$.[6] After production is completed, both k and z fully depreciate. The marginal rate of transformation of final goods into k and z equals 1.

3.1. Financial frictions

The lending process is distorted by a friction resulting from the possibility of default by entrepreneurs. In particular, the following assumptions are made. First, in the initial stage of production, an entrepreneur can default on her debt, close the firm, and bring with her all intangible capital z she has accumulated. In this case, the lender will seize the physical capital that has been pledged for the loan. After defaulting on the debt, the entrepreneur can open a new firm, borrow funds from another lender, and start production (of course, she might choose to default again). Second, after production is completed, the lender can seize output y, provided the original firm has not been closed. This ensures repayment of the loan whenever production is completed.[7] Third, during the first stage of production, the lender can determine how the funds will be

[6] Note that both stages happen between periods $t+1$ and $t+2$.
[7] Note that capital fully depreciates so it cannot be used to ensure repayment after production.

utilized by the borrower. This captures the fact that lenders frequently monitor the use borrowers make of loaned funds, and covenants specify the allocation of the resources they provide.[8]

Given these assumptions, it is clear that lenders will always require loaned funds to be completely invested in and backed by tangible capital. If any part of a loan was invested in intangibles, the entrepreneur would have the incentive to close the firm, keeping the intangible capital accumulated. She could then open a new firm with another lender who would lend to her against physical capital only. When the entrepreneur receives a loan backed by tangible capital, she has no incentive to default before production, because the tangible capital would be seized by the lender. Nor has she incentive to default after production occurs, because the lender can seize the output of production. Finally, note that competition among lenders implies that the return on these loans equals the marginal productivity of tangible capital. Under these conditions, the following result holds:

Lemma 1. *Borrowing requires collateral in the form of physical capital. In other words, before investing, each middle-aged entrepreneur of generation t faces the borrowing constraint* $d_t(t+1) \le k_t(t+1)$.

Hence, the problem of a middle-aged entrepreneur of generation t can be written as

$$\max_{(k,z,l,d)} \{y_t(t+2) - R(t+2)d_t(t+1) - w(t+2)l_t(t+1)\}$$

subject to

$$k_t(t+1) + z_t(t+1) \le w(t+1) + d_t(t+1)$$

$$d_t(t+1) \le k_t(t+1)$$

Since the first constraint always binds in equilibrium, the borrowing restriction can be rewritten as $z_t(t+1) \le w(t+1)$. Simply put, the possibility of default together with the lack of pledgeability implies that intangible capital is financed with internal funds only.[9]

It is useful to compare the assumptions above with the environment of other papers. Martin and Ventura (forthcoming), for example, prevent borrowing by the agents with better investment opportunities relative to their peers due to an "unspecified market imperfection". Lorenzoni (2008) introduces limited risk-sharing by assuming that entrepreneurs can default *after* they produce, losing part of the output and the remaining capital. With limited pledgeability of only one type of capital, in our model the default decision must be allowed to happen when K and Z are still differentiated. This is similar to Kiyotaki and Moore (1997), where the borrower can close the firm and take away his human capital at any time, leaving behind the land (the tangible part) to the lender.

4. Equilibria in the bubbleless economy

This section proves the existence and characterizes the bubbleless equilibria of the model. In what follows, aggregate variables are denoted by upper case letters. For instance, the aggregate output of generation t is given by $Y_t(t+2) = \pi y_t(t+2)$. Moreover, because labor is supplied inelastically every period, the individual labor supply is normalized to 1 and omitted from now on. A bubbleless competitive equilibrium is defined as follows:

Definition. The sequence of allocations $\{l_t(t+1), k_t(t+1), z_t(t+1), d_t(t+1)\}_{t=0}^{\infty}$ constitutes a dynamic bubbleless competitive equilibrium of the economy if there exists a sequence $\{w(t+1), R(t+1)\}_{t=0}^{\infty}$ of price vectors such that, at each time t, agents maximize their objectives and the labor and savings markets clear.

Solving in closed form for the equilibrium is possible because of the simplifying assumptions introduced before. When young, agents supply labor inelastically. When middle-aged, households supply their savings to the entrepreneurs inelastically as well.[10] Only entrepreneurs take active decisions, which coincide with profit maximization for the firm. Therefore, the equilibrium wage and interest rate are determined by the F.O.Cs of the firm:

$$w(t+2) = (1-\alpha-\beta)\frac{y_t(t+2)}{l_t(t+1)} = (1-\alpha-\beta)Y_t(t+2) \tag{3}$$

$$R(t+2) = \alpha\frac{y_t(t+2)}{k_t(t+1)} = \alpha\frac{Y_t(t+2)}{K_t(t+1)} \tag{4}$$

where $K_t(t+1) = \pi k_t(t+1)$.

[8] Often, loans are explicitly tied to particular uses, as in the case of commercial or residential mortgages.

[9] In our model, entrepreneurs cannot issue equity in order to invest, but are constrained to issue collateralized debt only. The crucial aspect here is not the distinction between debt and equity financing, but rather the asymmetry between inside and outside investors. In this respect, physical capital works as a mechanism through which managers commit not to expropriate investors, a property that cannot be mimicked by intangible capital.

[10] The model can also be solved in closed form if agents have log preferences over consumption in the last two periods of life. All the results derived below are valid within this more general context, apart of a renormalization of parameters, as shown in the Appendix. Note that the assumption of log utility not only introduces a meaningful savings decision: it also implies that the agents are risk averse. This makes it difficult to derive closed-form solutions for the case of stochastic bubbles, but calculations in the Appendix show that even in this case the main results still hold.

Two markets are open in period $t+1$. One is the market for labor, in which the young of generation $t+1$ supply labor to generation t's firms. Its equilibrium is given by $\pi l_t(t+1)=1$. The other market is the one for savings which achieves equilibrium when

$$K_t(t+1)+Z_t(t+1)=w(t+1)=(1-\alpha-\beta)Y_{t-1}(t+1) \tag{5}$$

The borrowing constraint (whose Lagrangian multiplier is λ_t referring to generation t) coupled with the optimal choice of intangible capital by entrepreneurs implies:

$$IRR(t+2)\equiv R(t+2)+\lambda_t=\frac{\beta y_t(t+2)}{z_t(t+1)} \tag{6}$$

$$\lambda_t[z_t(t+1)-w(t+1)]=0 \tag{7}$$

$$\lambda_t \geq 0 \tag{8}$$

These conditions show that, whenever the financing constraint binds, the marginal productivity of intangible capital is higher than the interest rate prevailing in the market for funds, which in equilibrium equals the marginal productivity of physical capital. The rate of return of intangibles is denoted IRR, to stress that it is the return an additional dollar could generate if freely transferred to an entrepreneur. Note that

$$IRR(t+2)-R(t+2)=\lambda_t \tag{9}$$

so λ_t captures the spread between the return on a unit of internal funds and the return on a unit of funds raised through collateralized debt.

4.1. Existence and characterization of the equilibrium

In order to characterize the behavior of the economy along an equilibrium path, it is necessary to determine under what conditions the borrowing constraint binds. Fortunately, the model features an interesting separation of the parameter space into two mutually exclusive and complementary regions, one where the constraint always binds and another where it never does. The next definition sets the stage for the formal proof of this result.

Definition. Let $\Theta=\{(\alpha,\beta,\pi,n):(\alpha,\beta,\pi)\in(0,1)\times(0,1)\times(0,1), 0<\alpha+\beta<1, 0<n\}$ be the parameter space in the model. Define two subsets which constitute a partition of Θ: $B=\{(\alpha,\beta,\pi,n)\subset\Theta:\beta/\alpha\geq\pi/(1-\pi)\}$ and $NB=\{(\alpha,\beta,\pi,n)\subset\Theta:\beta/\alpha<\pi/(1-\pi)\}$.

Proposition 1 builds on this definition to analyze the existence of a dynamic bubbleless equilibrium.[11] It is straightforward to show that, if a steady-state exists, all real variables must grow at the rate $(1+g)\equiv(1+n)^{1/(1-\alpha-\beta)}$.

Proposition 1. *There always exists a trivial equilibrium in which K, Z, and Y are equal to zero. Moreover, if $(\alpha,\beta,\pi,n)\in B$, the financing constraint is binding along all equilibrium paths; the economy has a unique non-trivial bubbleless globally stable steady-state (where the borrowing constraint binds). If instead $(\alpha,\beta,\pi,n)\in NB$, the financing constraint is slack along all equilibrium paths; the economy has a unique non-trivial bubbleless globally stable steady-state (where the borrowing constraint is slack).*

According to Proposition 1, for a given value π, as the ratio β/α increases, the economy moves from equilibria in which financing constraints are irrelevant to ones in which they bind. This occurs because, other things equal, the demand for intangible capital is increasing in β. The higher the ratio β/α, the more entrepreneurs want to accumulate intangible capital instead of physical capital. After a certain point, all internal funds are invested in Z, and financing constraints become binding. At this point, the relative supply of stores of value is distorted because households' savings can only be allocated to physical capital despite the higher productivity of intangibles. The over-accumulation of K reduces its marginal productivity and the rate of interest in the market for funds, which is key for rational bubbles to exist.

5. Bubbly equilibria

The results in Section 4 provide a framework for the analysis of bubbly equilibria. A rational bubble is defined as a security in unit supply which lasts forever, pays no dividends, and has price $B(t)>0$ at time t. Moreover, any individual can trade this security in the market.

The presence of a bubble changes some of the equilibrium conditions outlined above. In particular, the savings market equilibrium requires that, at each period, there are enough savings to finance physical and intangible capital accumulation as well as the acquisition of the bubbly security:

$$K_t(t+1)+Z_t(t+1)+B(t+1)=(1-\alpha-\beta)Y_{t-1}(t+1) \tag{10}$$

[11] While analyzing equilibrium allocations, we focus exclusively on the aggregate variables (Y,K,Z) and on the prices (w,R). Knowing the path of these aggregates is sufficient to fully describe the economy, since all other variables are immediately derived from them.

Because any individual can purchase the bubbly security in the market, its rate of return – the rate at which its price evolves over time – is equalized to the return on the worst investment opportunities available to investors. This is always given by the return R that households obtain on their savings. Therefore, in any dynamic equilibrium the value of the rational bubble evolves according to: $B(t+1)=R(t+1)B(t)$. In case financial frictions are binding, entrepreneurs will never be interested in the bubble, since their funds are invested at the rate *IRR* which is strictly larger than R. Hence, in any constrained equilibrium, the bubble is held in its entirety by the households of each generation.

5.1. Characterization of bubbly equilibria

Define $\hat{Y}_t(t+2)\equiv Y_t(t+2)/A(t+2)^{1/(1-\alpha-\beta)}$, $\hat{K}_t(t+1)\equiv K_t(t+1)/A(t+2)^{1/(1-\alpha-\beta)}$ and $\hat{Z}_t(t+1)\equiv Z_t(t+1)/A(t+2)^{1/(1-\alpha-\beta)}$. Moreover, let B^* represent the bubble-to-output ratio, i.e., $B^*(t+1)\equiv B(t+1)/Y_{t-1}(t+1)$. The first three terms formalize the usual concept of a variable per efficiency unit, while the last one defines the bubble per unit of output. Then:

Lemma 2. *The vector $[Y_{t-1}(t+1),B(t+1)]$ is a state vector for the economy. The vector $[\hat{Y}_{t-1}(t+1),B^*(t+1)]$ is a state vector as well, given an initial value of technology $A(0)$. In addition if, at time t, $[\hat{Y}_{t-1}(t+1),B^*(t+1)]$ belongs to a dynamic equilibrium where $(\alpha,\beta,\pi,n)\in NB$, the financing constraint is slack in the next period. Alternatively if, at time t, $[\hat{Y}_{t-1}(t+1),B^*(t+1)]$ belongs to a dynamic equilibrium where $(\alpha,\beta,\pi,n)\in B$, there exists a threshold $B^*_{max,b}$ such that for $B^*(t+1)\le B^*_{max,b}$ the financing constraint is binding in period $t+1$, while for $B^*(t+1)>B^*_{max,b}$ the financing constraint is slack in period $t+1$.*

If the economy is in the *NB* region, the financing constraints are slack along any equilibrium path. The case where the economy belongs to the *B* region is more involved. Intuitively, for sufficiently small bubbles, the financing constraint is binding, since the bubble does not drain too many funds. In this case the stock of capital will be high, inducing entrepreneurs to fully invest in intangibles. On the other hand, if the bubble is sufficiently large, households will finance a small amount of physical capital. In this case, entrepreneurs will invest in K as well, which means that the borrowing constraint becomes slack.

Based on that, the bubbly dynamic equilibria can be characterized in detail. Consider first Proposition 2, which restates Tirole's (1985) result.

Proposition 2 (*Tirole, 1985*). *If the bubbleless steady-state features $\pi IRR+(1-\pi)R<(1+g)$ there exist dynamic bubbly equilibria.*

Whenever the average rate of return of the economy is below its growth rate in the bubbleless steady-state, rational bubbles can exist. This well known condition has been traditionally interpreted as a sign of dynamic inefficiency, since a low rate of return results from excessive accumulation of capital. It is important to note that this proposition defines the necessary and sufficient conditions for the existence of bubbly equilibria in the *NB* region—where R is equalized to *IRR*. In the *B* region, on the other hand, it is possible for bubbly equilibria to survive even if the bubbleless steady-state features $\pi IRR+(1-\pi)R\ge(1+g)$ as shown by Farhi and Tirole (forthcoming).

It turns out that in the bubbleless steady-state, the comparison between the average rate of return and the growth rate of the economy is equivalent to the AMSZ test about the productivity of the corporate sector. AMSZ compare, at every period, the amount of funds absorbed by the corporate sector in the form of capital expenditures versus the excess of its output over labor costs. The authors present strong evidence that the corporate sectors of OECD economies are highly productive since they generate more funds than what they absorb. We require our model to satisfy this empirical benchmark.

Lemma 3. *In any bubbleless equilibrium, the economy satisfies the AMSZ test if and only if $\alpha+\beta\ge\frac{1}{2}$, which is equivalent to a bubbleless steady-state where $\pi IRR+(1-\pi)R\ge(1+g)$.*

5.2. Bubbly equilibria in the binding region

The following paragraphs characterize the bubbly equilibria in the *B* region subject to the AMSZ empirical benchmark, which, for our purposes, defines the relevant region of the parameter space.[12] The text omits the discussion of the dynamic equilibria in the other regions of the parameter space, since they can be derived in similar ways.

Proposition 3. *In the B region with $\alpha+\beta\ge\frac{1}{2}$, there exist dynamic bubbly equilibria that converge to a steady-state in which $B^*=(1-\alpha-\beta)(1-\pi)-\alpha$, and the borrowing constraint is binding at all times, as long as $B^*>0$. There also exist dynamic equilibria such that the bubble-to-output ratio is always non-negative but converges to zero over time.*

Fig. 2 represents graphically the relevant regions of the parameter space in terms of α and β, for given π and n. The *B/NB* line has slope $\pi/1-\pi$, and divides the space Θ into the *B* and *NB* regions.

[12] This restriction implies a counterfactually small share of labor income relative to output. However, this results from some of the stark assumptions introduced before to make the model tractable and are not directly linked to the most important aspects of the analysis. For instance, our assumption of full depreciation of capital is partly responsible for this difficulty.

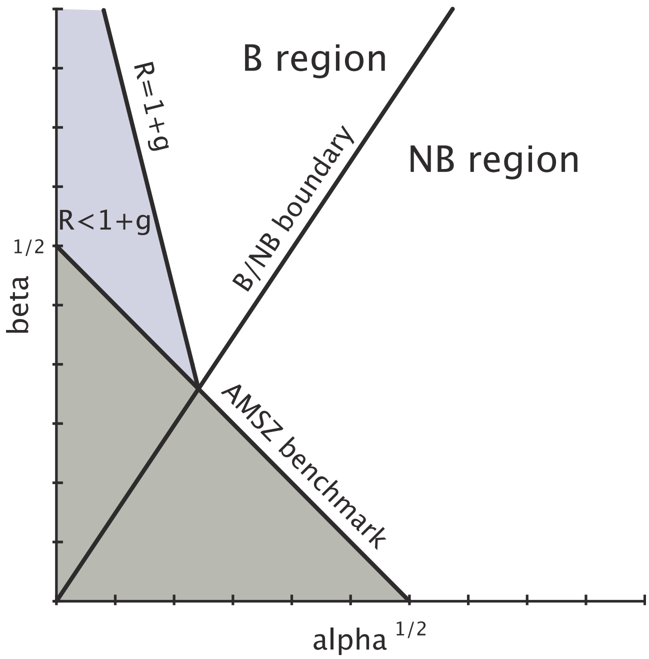

Fig. 2. Characterization of the parameter space, for the parameters of the production function $y = Ak^{\alpha}z^{\beta}l^{1-\alpha-\beta}$. k is tangible capital and z is intangible capital. g and R are respectively the growth rate of the economy and the interest rate on loans in the steady-state. The B region is the region in which financing constraints are binding in the bubbleless equilibrium. The NB region is the region in which financing constraints are not binding in the bubbleless equilibrium. The shaded area is the region in which rational bubbles can be sustained in the long-run. The dark portion of the shaded area is the region in which the economy does not satisfy the AMSZ test for dynamic efficiency.

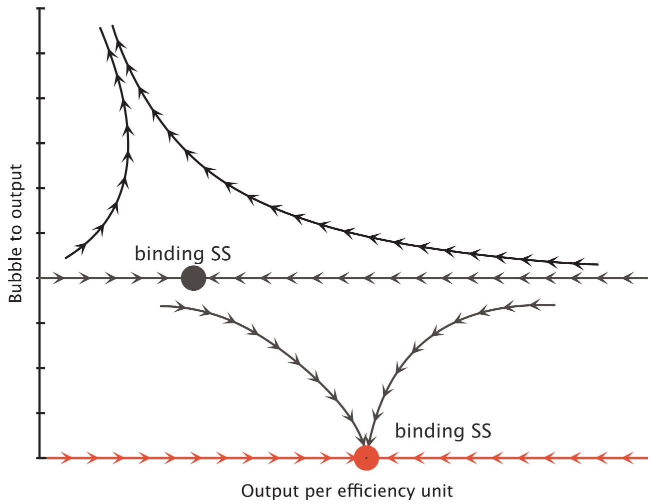

Fig. 3. Bubbly and bubbleless equilibria of the economy, in terms of normalized output and bubble-to-output ratio. Large dots are steady states. Bubbleless equilibria lie on the x-axis. Above, we plot equilibria that converge to the bubbleless steady-state (the bubble disappears asymptotically); equilibria that converge to a bubbly steady-state; the paths pictured above those, in which the bubble-to-output ratio explodes, do not constitute equilibria for this economy.

Proposition 3 extends and qualifies the result of Proposition 1 – about bubbleless equilibria – to bubbly equilibrium paths, by identifying an additional division of the parameter space, related to the AMSZ line. This line separates economies in which the AMSZ benchmark is satisfied ($\alpha + \beta \geq \frac{1}{2}$) from those in which it is not. The dynamics of the bubbly equilibrium paths can be described as follows. In the region below the AMSZ line, all of Tirole's (1985) standard results apply, whether the constraint binds or not. There can be bubbles that disappear asymptotically and bubbles that grow forever together with the economy. This region, however, is not considered in the rest of the paper, since it contradicts the empirical evidence.

In the area above the AMSZ line, the separation between NB and B defines the dynamics of the bubbly equilibria, just like in the case of the bubbleless equilibria. In the shaded area of the B region, the bubbleless steady-state features

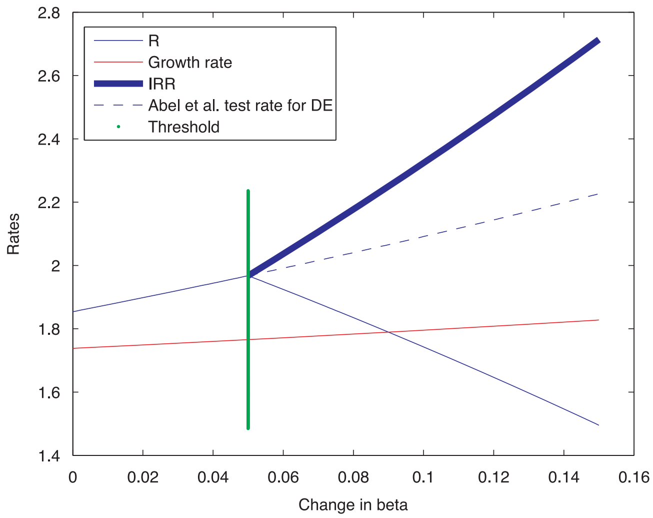

Fig. 4. Example of the effect of a technological change (increasing β and decreasing α) on steady-state interest rate (R), internal rate of return of intangible capital (IRR), and growth rate of the economy. The dotted line represents the benchmark for the standard AMSZ test of dynamic efficiency. The vertical line represents the point in which technological progress makes financial frictions bind.

$R<(1+g)$. This condition guarantees that there exist equilibrium paths in which the bubble-to-output ratio is positive but disappears asymptotically, as well as equilibrium paths that converge to a steady-state with a positive bubble-to-output ratio. In all bubbly paths the financing constraints are binding. In the rest of the B region and in the whole NB region, instead, no rational bubbles can be sustained in equilibrium.

Fig. 3 represents the dynamics of the economy in the B region, above the AMSZ line, for $B^*>0$. The lighter path along the x-axis represents the bubbleless equilibrium path, that converges to the bubbleless steady-state (lighter dot on the x-axis). All the other equilibrium paths have a positive bubble. They either converge to the bubbleless steady-state (the bubble disappears asymptotically), or they reach the bubbly steady-state through a saddle path, where the bubble is constant as a fraction of output.

The previous results can be easily extended to the case of stochastic bubbles. A full discussion in the Appendix shows that the results and main intuition would be valid in an environment where, at every period, the rational bubble survives with probability p or bursts with probability $1-p$ as in the classic model of Blanchard and Watson (1982). Moreover, it shows that, in a slightly modified version of the model where all generations of entrepreneurs can issue new amounts of the bubble, its bursting can induce a long-term contraction in output, matching the historical evidence.

6. Intangible capital, technological change, and bubbles

Section 5 shows that economies which satisfy the AMSZ condition can sustain bubbles only if intangible capital is sufficiently important in the production process. With a high β relative to α, financing constraints are active, and the equilibrium interest rate is low—which allows rational bubbles to be sustained.

To better illustrate the point, we calibrate the model and calculate the steady-state rates of return and growth rate of the bubbleless economy for different values of α and β.[13] In this exercise, the fraction of income absorbed by intangible capital β is increased by 15 percentage points, while α is decreased by 12 percentage points. The remaining 3 percentage points are taken from labor in order to preserve constant returns to scale. This calibration mimics the change in the share of output accruing to the different factors in the United States in the last 30 years (Corrado et al., 2009). Fig. 4 shows how the bubbleless steady-state rates R and IRR shift as β gradually increases and α decreases. The horizontal axis represents the change in β. It also compares the growth rate of the economy $1+g$ with the equilibrium average rate of return, $(1-\pi)R+\pi IRR$.[14]

The green vertical threshold separates the NB region (left) from the B region (right). For small values of β, the economy is in the NB region. The returns on tangible and intangible capital are equalized, so that $R=IRR$. Since the AMSZ condition is satisfied, $R>1+g$ in the steady-state, and no bubbles can exist. For larger values of β/α, on the other hand, the economy is in the B region. The wedge between R and IRR increases with β/α and might drive R below $1+g$ in steady-state if β is sufficiently high. After this point, bubbly equilibria are possible.

The change in β/α calibrated here represents a comparative statics exercise, but its implications are identical to an unexpected and exogenous technological shift in the economy. More specifically, suppose that the economy is originally in

[13] Further details of the calibration are reported in the Appendix.

[14] Note that in our calibration the economy always satisfies the AMSZ test, since the average rate – the dotted line – is always above $1+g$.

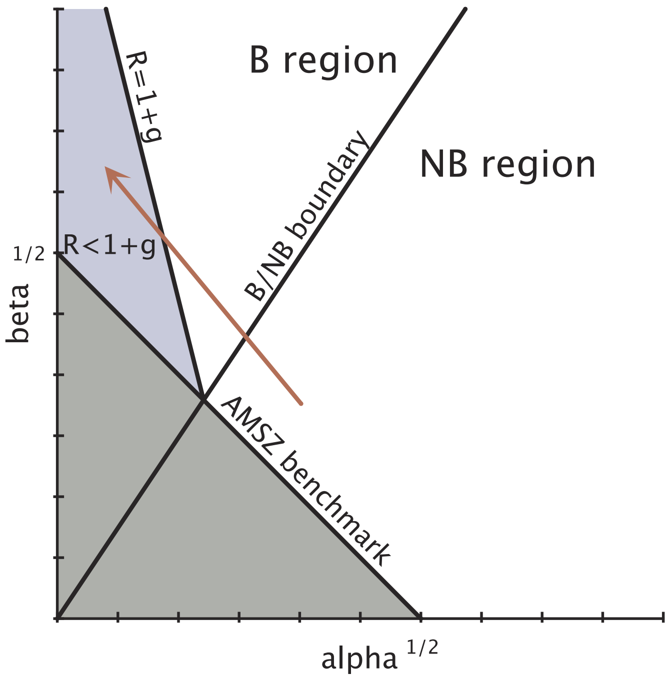

Fig. 5. Technological change (increase in β and decrease in α) represented in the parameter space. The change brings the economy from the *NB* region to the shaded area of the *B* region in which bubbles can arise. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

the *NB* region of the parameter space, and assume that it satisfies the AMSZ condition, so no bubble can exist. Suppose all agents originally expect technology to be constant forever, but, at a certain time *T*, (α,β) suddenly shifts to (α',β'). Assume that $(\alpha'+\beta')>0.5$, ensuring the new specification satisfies the AMSZ condition as well. Finally, suppose no more changes are due. The arrow in Fig. 5 represents this shift in the space of (α,β). Under these assumptions, the following result holds:

Proposition 4. *For any value of* $(\alpha'+\beta')\in(0.5,1)$, *there exist parameters* (α',β') *with* β'/α' *sufficiently large such that a bubble can be created at the beginning of T. Before time T, there can be no bubbles.*

In general, at time *T* the bubble could be created by consumers and entrepreneurs. The most interesting case to explore is that in which the entrepreneurs of generation $T-1$ issue the bubble, which allows them to expand investment. In particular, the issuance of a bubble by the entrepreneurs has three effects. First, it immediately alleviates the financing constraints of the existing entrepreneurs, reducing distortions in capital allocation and directly benefiting all individuals from generation $T-1$. Second, there is a positive spillover impact on future generations since the increased production raises labor income, relaxing future financing constraints as well. Because of decreasing returns to capital, though, its importance dies out over time. Finally, there is a competition effect: as the bubble grows, it competes for funds with entrepreneurs, with a negative impact on the accumulation of capital and production by future generations.

6.1. *Stochastic technological progress and bubbles*

The analysis above can be extended to investigate the case in which agents anticipate the possibility of technological change. In what follows, it is shown that the *possibility* of an increase of β relative to α is already sufficient to allow for bubbly equilibria. Interestingly, bubbles can arise even if the economy is originally in the *NB* region and if it satisfies the AMSZ test at all times.

Consider an economy that initially belongs to the *NB* region of the parameter space and also satisfies the AMSZ condition. At every period, there is a probability $q>0$ that technological progress will take place, changing the original parameters of the production function (α,β) to new values (α',β'). Once this happens, no further changes occur and the economy becomes completely deterministic. The new specification (α',β') belongs to the *B* region and also satisfies the AMSZ criterion: $(\alpha'+\beta'>0.5)$. Additionally, suppose that entrepreneurs know exactly their technology at the time they make investment decisions. Hence, if technological change has not taken place until the beginning of period $t+1$, for example, the middle-aged entrepreneurs of generation *t* will produce with the old technology. In this case, uncertainty can only affect the entrepreneurs of future generations. Under these assumptions, the following result holds:

Proposition 5. *For any value of* $(\alpha'+\beta')\in(0.5,1)$, *there exist parameters* (α',β') *with* β'/α' *sufficiently large such that bubbly equilibria exist.*

In the situation described above, the probability of the economy experiencing technological progress is positive and constant at all times until the change takes place. As a consequence, technological progress occurs with probability 1 in the long-run. If β'/α' is sufficiently large, thus, the steady-state will feature low interest rates almost surely. It can be shown, however, that bubbles may arise even if there is uncertainty about interest rates in the long-run. To see that, assume a similar model as before. Suppose, however, that the permanent technological progress occurs with probability $q>0$ at the very beginning of time T only, where T is a future period. For any other date, $q=0$ and there is no uncertainty. Then:

Proposition 6. *For any value of* $(\alpha'+\beta') \in (0.5,1)$, *there exist parameters* (α',β') *with* β'/α' *sufficiently large to allow for bubbly equilibria in which the bubble exists before time T. If technological change does not occur at date T, the bubble immediately bursts and the economy converges to the bubbleless steady-state thereafter. If, on the other hand, technological change occurs at date T, the bubble is positive at all times.*

Proposition 6 illustrates how the stochastic nature of bubbles might be purely linked to the randomness of technological progress. If there is a technological "disappointment" and the production structure is unchanged at time T, the bubble immediately bursts.

The intuition for the results in Propositions 4,5, and 6 is straightforward. Technological progress shifts the production function to be biased towards intangibles relative to physical capital. This moves the economy to a region of the parameter space where the bubbleless steady-state features binding financing constraints and low interest rates, despite a high *IRR*. This change, illustrated by the red arrow in Fig. 5, allows for the co-existence of bubbles and efficiency. Interestingly, the very possibility of such a change occurring is already sufficient to generate paths with a positive bubble even before the technological shift takes place.

6.2. Relation to the savings glut explanation

One recent alternative explanation for asset price bubbles is based on the so called "savings glut" phenomenon—see Bernanke (2005). It points to the fast development of emerging markets like China and oil-exporting countries, that have large amounts of savings and a shortage of stores of value. The excess funds from these countries have flooded the capital markets of developed nations, putting downward pressure on interest rates and stimulating the formation of speculative bubbles. Besides identifying a separate underlying story for the emergence of bubbles in developed economies, our model differs from the savings glut explanation along two main dimensions.

First, our model hinges on a *relative* imbalance of high-yield versus low-yield assets in developed economies – a surplus of K and a shortage of Z – rather than on an absolute lack of sound stores of value. While the latter is certainly reasonable for developing countries, it seems less relevant for developed nations. Second, most of the models that attribute the emergence of rational bubbles to an increase in the supply of funds feature a single-factor economy (physical capital), with linear technology (*AK*) and limited pledgeability of output. The linearity assumption, however, is not innocuous. The limited pledgeability of output slows the accumulation of capital resulting from the inflow of foreign funds, but does not eliminate it. In an economy where physical capital has decreasing marginal returns, this accumulation would in fact *lower* the internal rate of return of funds. At some point, thus, the economy would have a sufficiently low productivity of capital to fail the *AMSZ* test, rendering it inconsistent with the available evidence. The assumption of linear technology shuts down this channel and keeps the internal return on funds artificially high.

In our framework, however, the savings glut explanation reinforces the technology-based approach. The separation between the two types of capital allows the accumulation of physical capital to actually *increase* the wedge between R and *IRR*. To the extent that foreign investors cannot overcome the moral hazard frictions in the domestic economy, an inflow of funds from abroad increases the stock of K and pushes the *IRR* up. This allows the *AMSZ* condition to be satisfied even for very low interest rates.[15] This intuition is formalized through a stylized example of an open economy presented in the Appendix.

7. Dynamic efficiency

As discussed in Sections 5 and 6, bubbly equilibria can exist even if the economy satisfies the AMSZ empirical benchmark. In AMSZ's original paper as well as in Tirole (1985), this restriction corresponds to the condition that the equilibrium interest rate is above the growth rate of the economy in the long-run. This has been traditionally interpreted as a sign of dynamic efficiency. In particular, in that setting the following three statements are equivalent: (i) the economy satisfies the AMSZ condition; (ii) the interest rate is higher than the growth rate of the economy in the long-run; and (iii) the economy is dynamically efficient.

In the presence of frictions, however, the equivalence of these statements is broken. For instance, Farhi and Tirole (forthcoming) have shown that statements (i) and (ii) can be delinked. We extend their analysis by proving that, in our

[15] Interestingly, for our effects to take place, there is no need for global savings to increase. As physical capital becomes less attractive due to technological change, even a stable amount of total savings is capable of pushing interest rates down. This is important since the evidence (Laibson and Moellerstrom, 2010) suggests that global investment has not increased as a fraction of global GDP since mid 1990s.

model, statements (i) and (iii) are not equivalent either. Moreover, the relation between statements (i) and (ii) is qualified. While these results are derived under a stylized setting, they illustrate a much more general point: financial constraints can decisively influence the validity of traditional tests of dynamic efficiency because they create heterogeneity in investment opportunities.

Lemma 3 (in Section 5) formalizes the connection between the AMSZ restriction and rates of return. Note that as long as $\alpha+\beta \geq \frac{1}{2}$, the AMSZ test will be satisfied. Proposition 7 formally proves that the AMSZ test is not sufficient to guarantee the efficiency of allocations once financial frictions are taken into account.

7.1. Pareto improvements

To understand efficiency, it is important to discuss the dimensions a planner could act on in order to achieve potential improvements over market allocations. First, whenever borrowing constraints bind, the planner can make intra-generational transfers from unconstrained households to constrained entrepreneurs. This, however, has no major implications for the topic of dynamic efficiency, since it does not address the issue of over or under-accumulation of capital. Much more interesting, though, is to consider improvements in the intertemporal margin while respecting existing borrowing constraints.

Definition. An allocation is constrained efficient if it cannot be Pareto improved by a planner who is not allowed to transfer resources from the unconstrained households to the constrained entrepreneurs within each generation.

Based on the original literature on dynamic efficiency, one could expect that the constrained efficiency requirement would coincide with the AMSZ test. The following proposition proves that this is not the case.

Proposition 7. *If financial frictions are binding, the AMSZ criterion is not a sufficient condition for constrained efficiency.*

The reason why the AMSZ criterion does not guarantee efficiency in our model is related to the heterogeneity of investment opportunities created by financial frictions. In the binding region of the parameter space, there is a wedge between the interest rate obtained by creditors and the rate of return earned by entrepreneurs. Looking at the average rate of return in the economy – as in the AMSZ test – is not sufficient to judge the efficiency property of market allocations, because there might be over-accumulation among those who cannot park funds on productive assets. The social planner can improve an allocation by transferring funds across households from different generations, compensating all entrepreneurs along the way. This can be done even if the growth rate of the economy is lower than the average rate of return, as long as it is greater than R.

One could argue, though, that the suggested transfer scheme does not do justice to the AMSZ test. On the one hand, the planner is not allowed to overcome the inefficiency resulting from moral hazard. On the other hand, she is allowed to identify the different groups according to their investment opportunities and target them for reallocation purposes. This criticism can be addressed by analyzing the efficiency properties of allocations that satisfy the AMSZ test in a setting where the planner cannot target specific groups.

Definition. An allocation is blind constrained efficient if it cannot be Pareto improved by a planner who can only transfer resources across generations without identifying households or entrepreneurs.

Interestingly, even when only blind reallocations are taken into account, it is still the case that the AMSZ benchmark does not coincide with efficiency, this time for the opposite reason: there exist allocations that do not satisfy the AMSZ benchmark, but cannot be improved by a blind social planner.

Proposition 8. *If financial frictions are binding, the AMSZ criterion is not a necessary condition for an allocation to be blind constrained efficient.*

The intuition behind this result is straightforward. The AMSZ test compares the average rate of return on the economy with its growth rate in the long-run. However, a planner trying to improve over market allocations who cannot target specific groups in society is required to transfer resources over time at the highest rate of return every period. This is the only way to ensure that those who face the best investment opportunities are not harmed by reallocations. Hence, the fact that the economy fails the AMSZ test does not imply there are feasible improvements for blind planners, unless $IRR < (1+g)$ in steady-state.

7.2. Bubbles and Pareto efficiency

The previous propositions have shown that the economy can satisfy the AMSZ benchmark while being inefficient. In addition, Sections 4 and 5 demonstrated that the economy can sustain bubbly equilibria even if the AMSZ condition is imposed. A natural question, therefore, is whether rational bubbles are capable of restoring efficiency. As in some of the previous literature (like Grossman and Yanagawa, 1993; Farhi and Tirole, forthcoming), we argue this is not the case in general, as the crowding out effect of the bubbles on investment has a negative externality on some agents.

To see that, consider an economy where borrowing constraints bind. In this context, the introduction of a rational bubble has redistributive effects that affect negatively some individuals. On the one hand, bubbles improve the investment opportunities of savers. On the other hand, as the bubble grows, it competes for funds with entrepreneurs who borrow resources to make real investments in future periods. Therefore, typical rational bubbles are not Pareto improving in our model.

8. Conclusion

The literature on asset shortages proposes that one of the structural causes of endless speculation in global financial markets is the inability of the world economy to properly supply the stores of value necessary to accommodate its increased wealth. Caballero et al. (2008a) point to the developing world as a source of imbalances in the supply and demand for assets. The present paper suggests that developed countries might have played a significant role in this process as well. Our analysis stresses that one important element favoring asset price bubbles in developed countries is not an absolute lack of stores of value, but rather a difficulty in generating high-yield assets relative to low-yield ones.

The mechanism described in this paper identifies one potential structural cause of bubbles, rooted on the interaction between technological progress and pledgeability of intangible capital. As economies develop and change, though, other forces may become more or less important at the same time. For example, the pledgeability of tangible capital itself may be much lower in developing economies than in developed ones—so that for developing countries that might be the main source of frictions. Naturally, even among advanced economies the degree of pledgeability of intangible capital will vary. As nations develop better instruments to transfer ownership of intangible capital (for example, a more efficient patent system), its effective pledgeability will improve, mitigating the frictions that, in our model, are at the source of rational bubbles. In fact, improving the pledgeability of assets might be the best response by policymakers for preventing and containing asset price bubbles in this environment. The interaction between technological and institutional progress and its implications for financial markets are interesting topics for future studies.

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Appendix A. Supplementary material

Supplementary data associated with this article can be found in the online version at http://dx.doi.org/10.1016/j.jmoneco.2012.03.004.

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